Review quiz

- What is the meaning of thermobaricity?

Review quiz

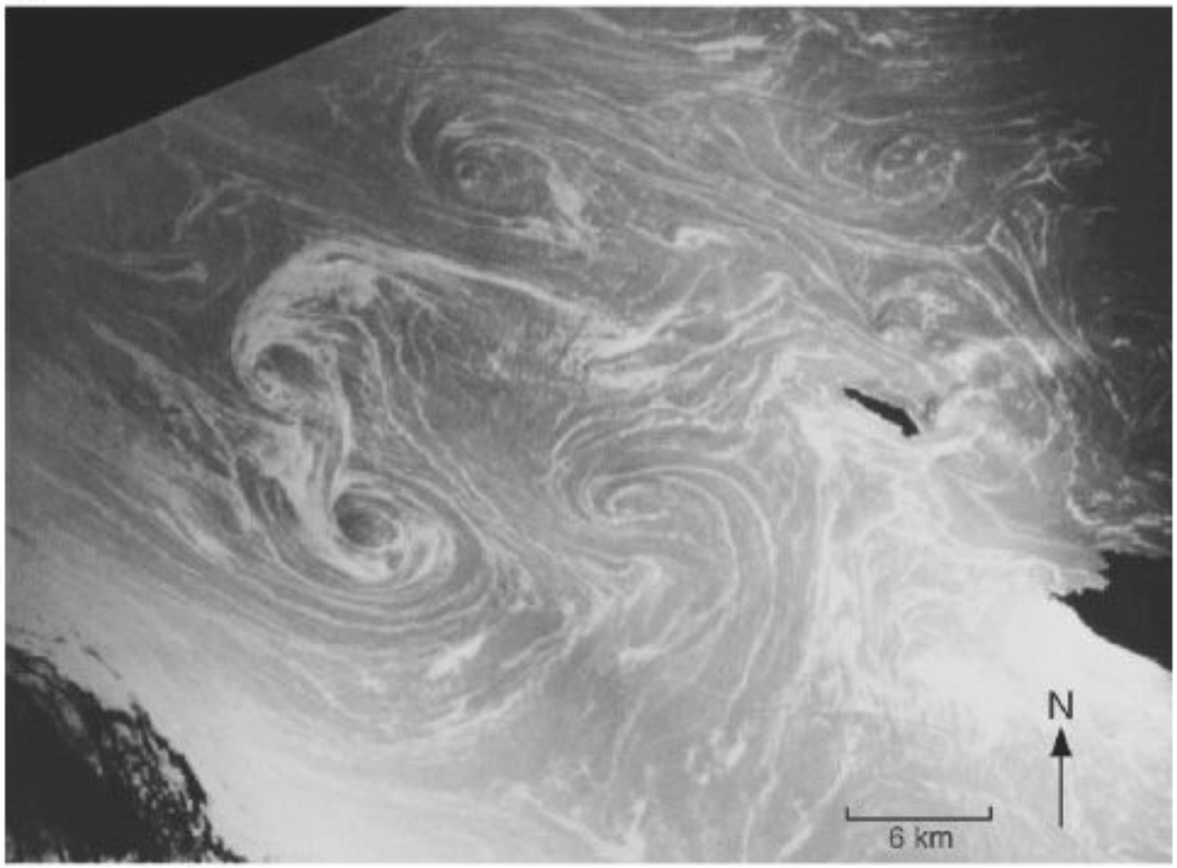
- What is the meaning of thermobaricity?

- Colder water is more compressible. In another words, density contour in T-S diagram becomes flatter under higher pressure (deeper depth)

Review quiz

- Which property of the seawater is NOT a conservative tracer?
  - (a) Density
  - (b) Potential temperature
  - (c) Salinity
  - (d) Neutral density

Week 3: Transport and mass balance



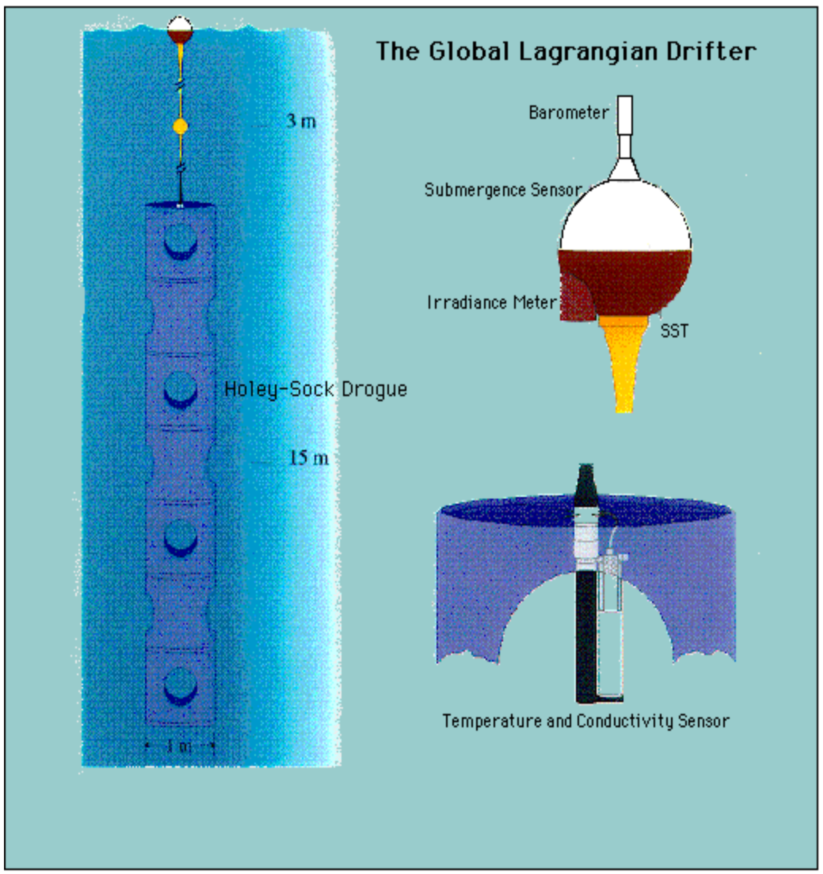
- Textbook: Chap 5.

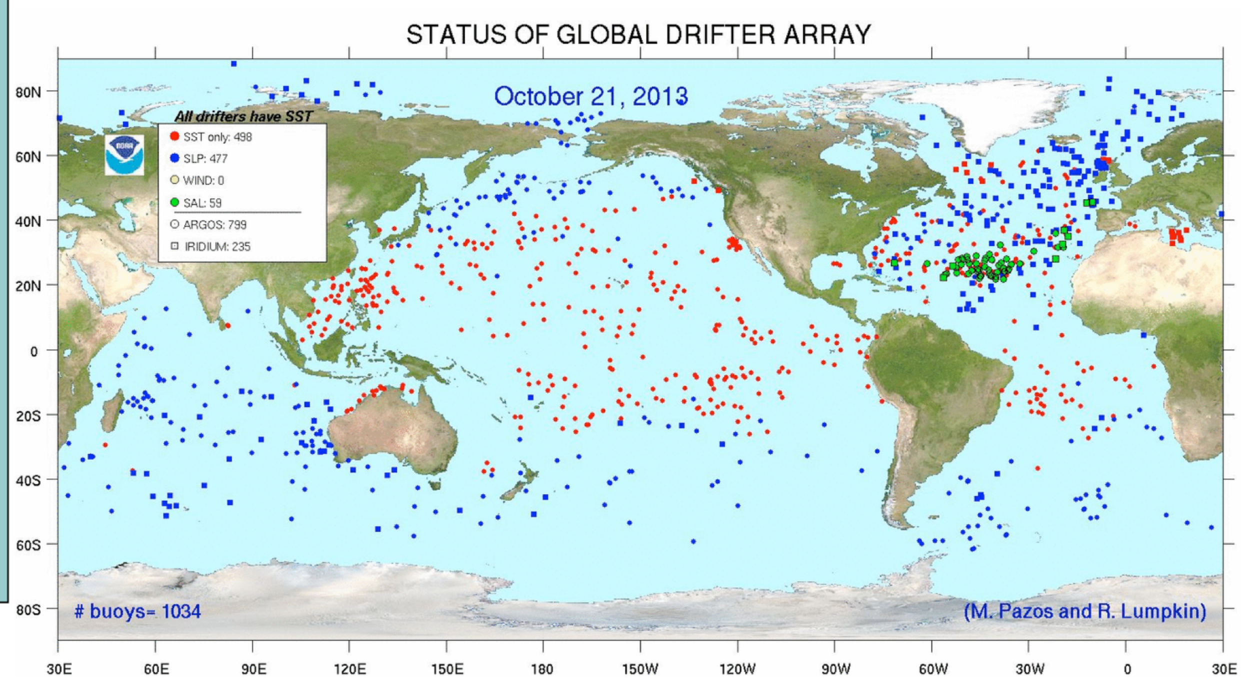
Lagrangian and Eulerian

- Lagrangian
  - Particle view of the fluid
  - Describes the trajectory of a specific water parcel
  - Obs platform: drifters, floats
- Eulerian
  - Field view of the fluid
  - Describes the motion at a specific location (x,y,z) and time (t)
  - Obs platform: ocean time series, mooring

Autonomous (Lagrangian) observation

Surface drifters: velocity and a few sensors (SST, SSS, air pressure are common)



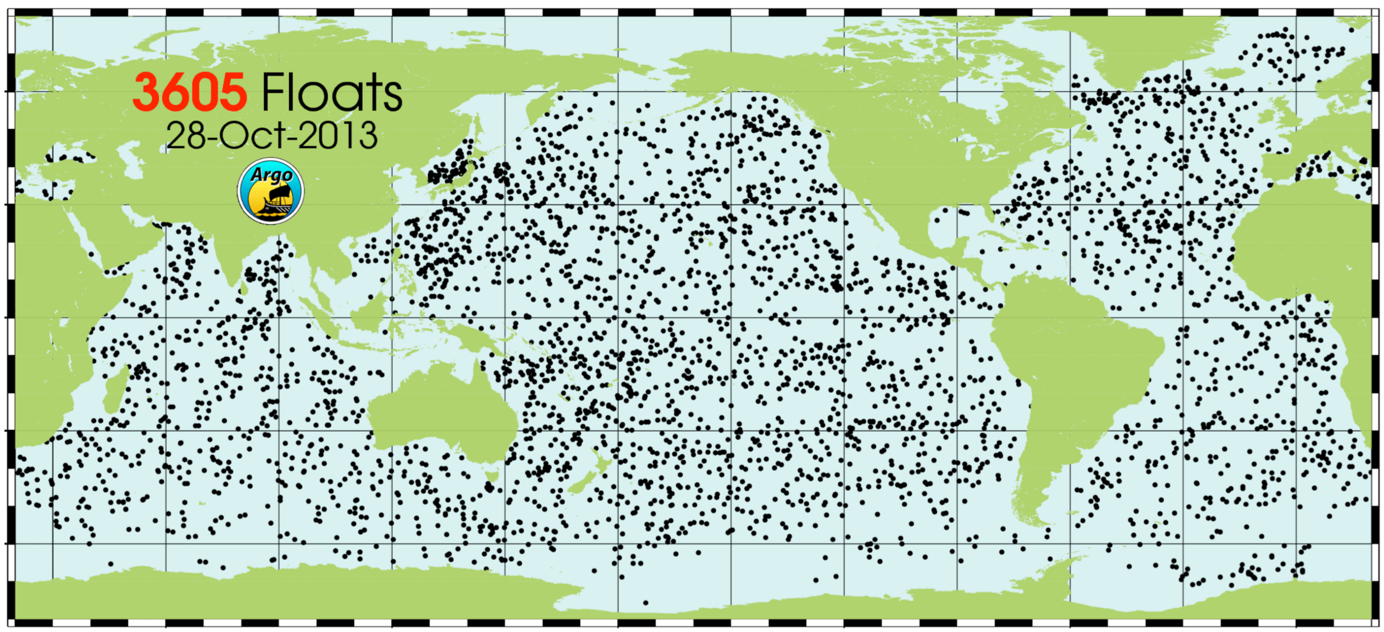


ARGO: subsurface, free-drifting floats

Profile to 2000 m and tracked every 5 to 10 days

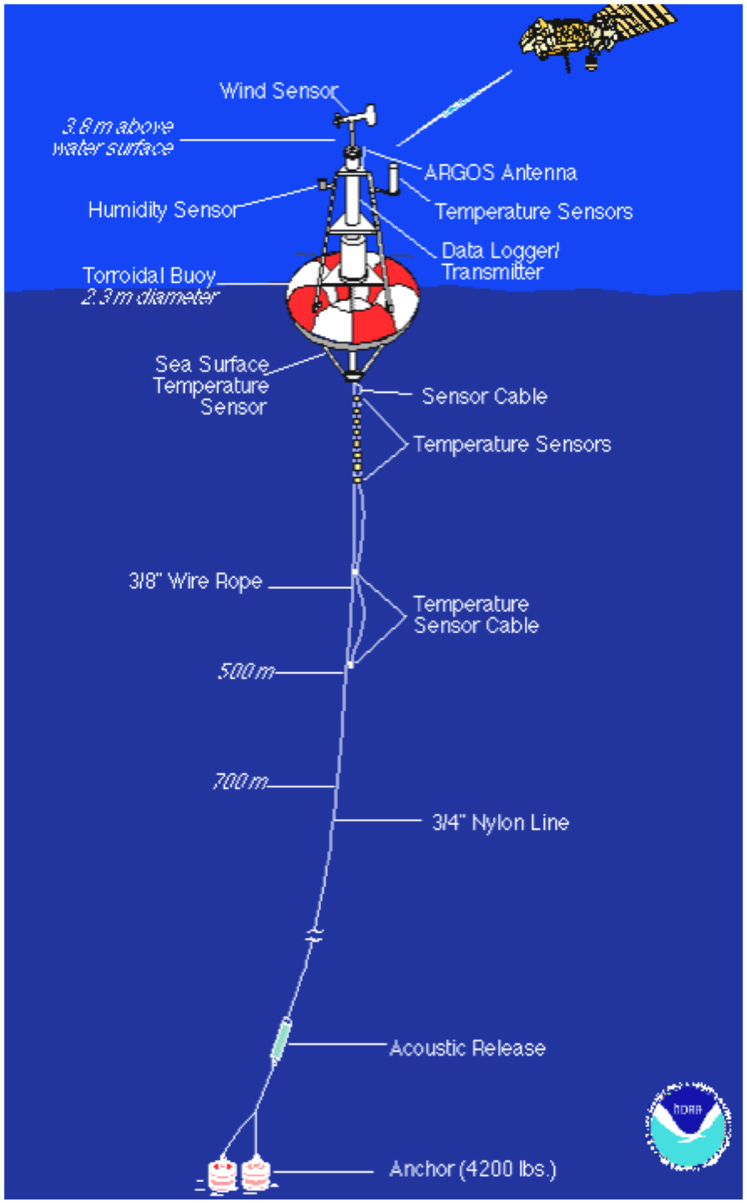
Best for repeat profiling of water column (core ARGO mission measures T and S only, but newer generation floats can also measure O_2, NO_3, pH, chl fluorescence)

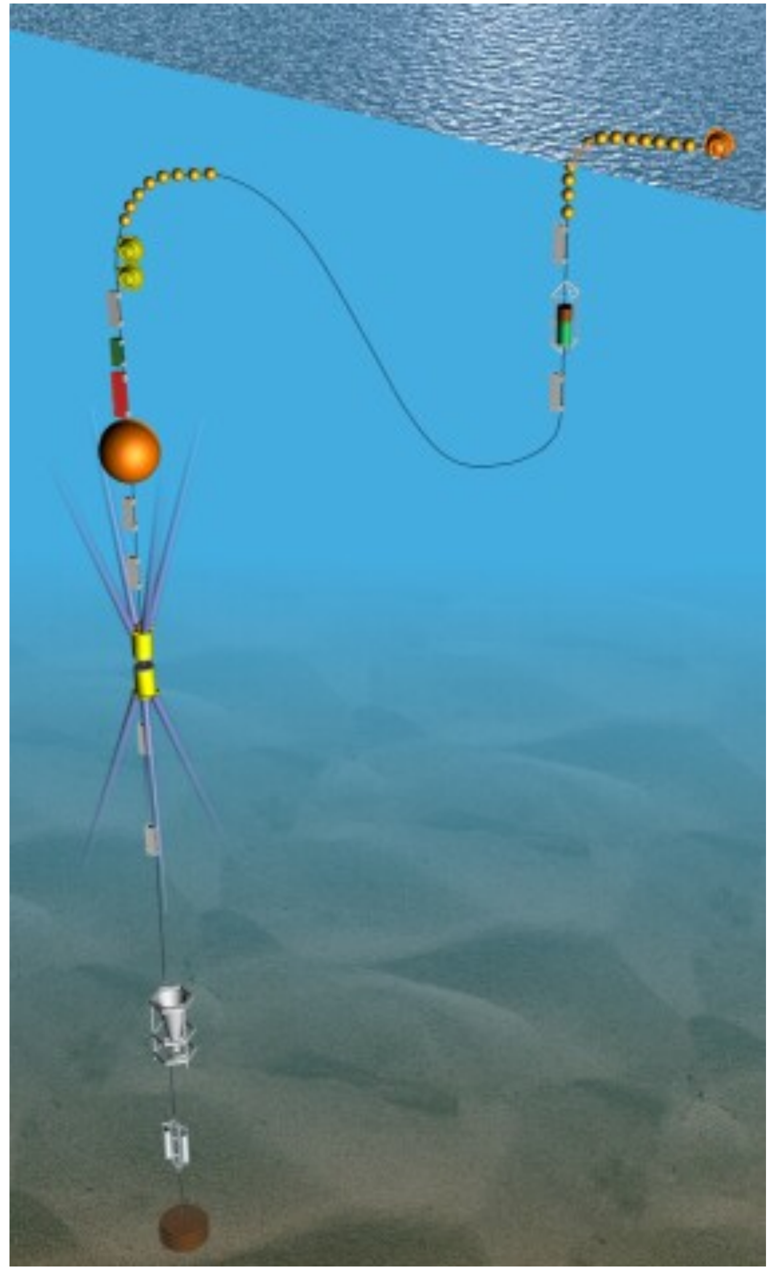
Up to 180 profiles for the life of battery cell



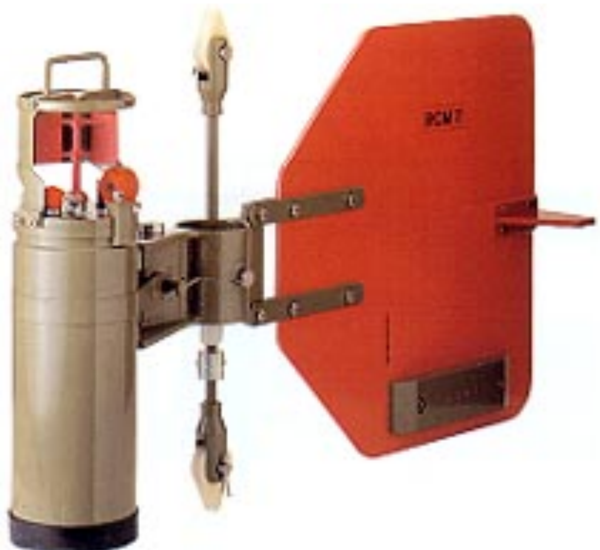
Mooring (fixed to bottom: Eulerian)

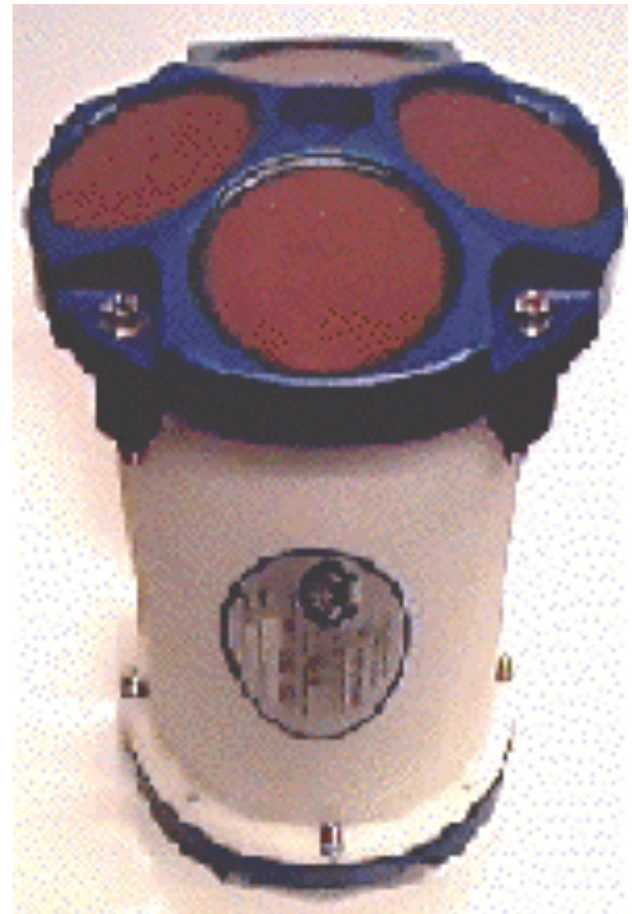
Moorings can sample with high rate, from surface to bottom, many simultaneous sensors, and can carry heavy instruments





Moored current meters (Eulerian)

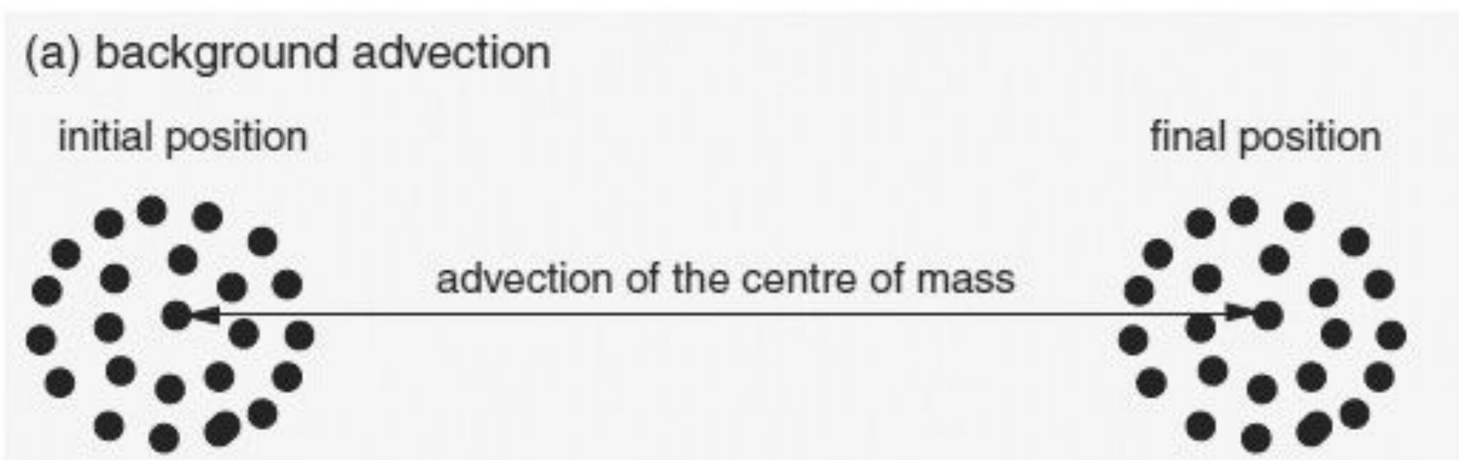




Acoustic Doppler Current Profiler (ADCP)

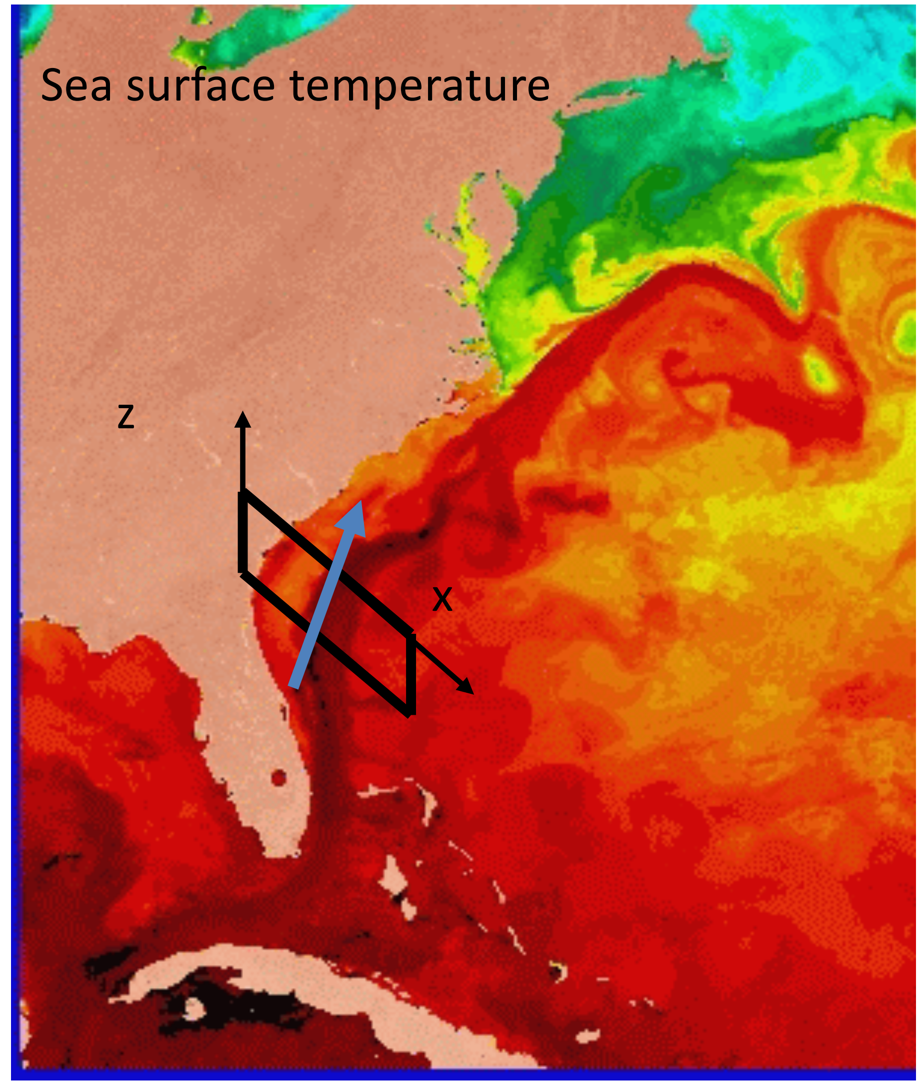
Advection

- Fluid properties (mass, T, S, nutrients, etc) moves with the flow of fluid (ocean currents)



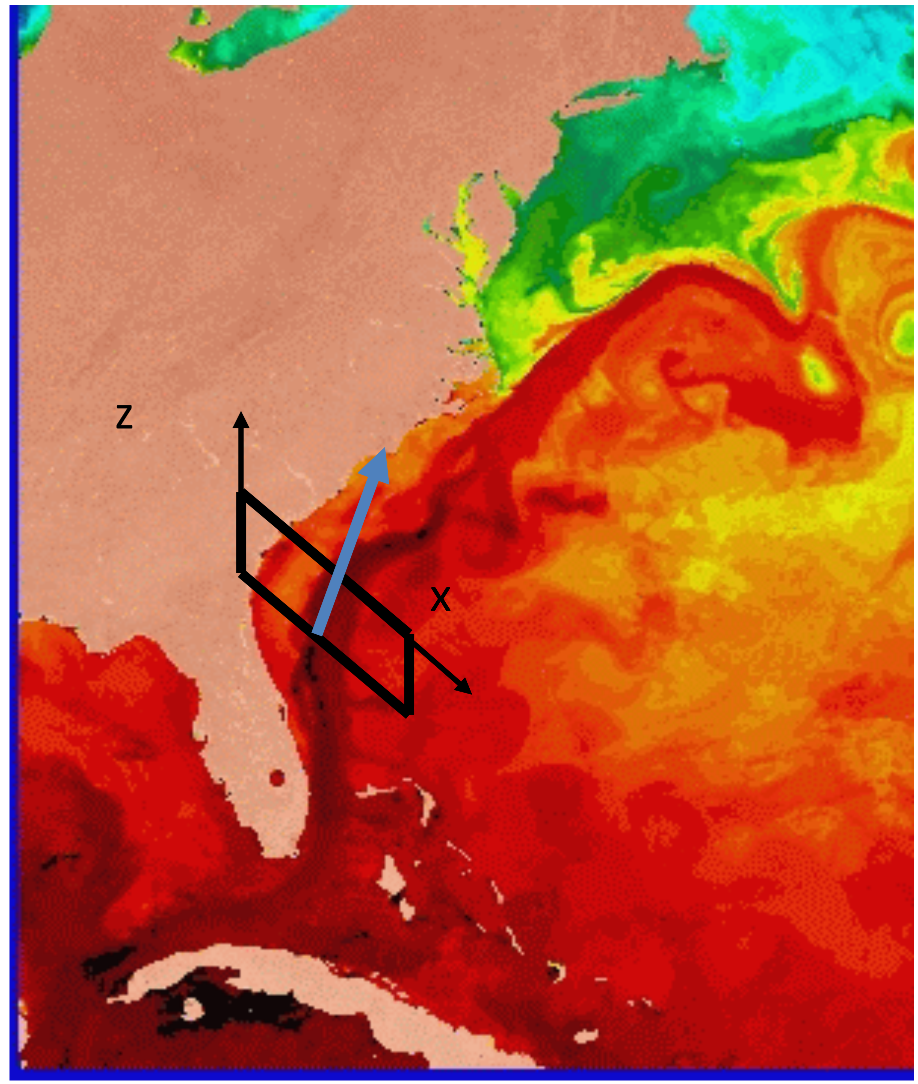
Quantify transport resulting from the gulf stream

- Gulf Stream “advects” volume, mass, warm water northward
- How much water, how much mass is carried by the G.S past a certain point ?
- Draw a vertical plane across the current (x,z are across-stream and vertical)
- Measure current velocity at each point in the plane, normal to the plane
- Compute volume transport (velocity times area) for each small location in the plane and add them up (integrate) for total transport through the cross-section



Quantify volume transport (advection) (example)

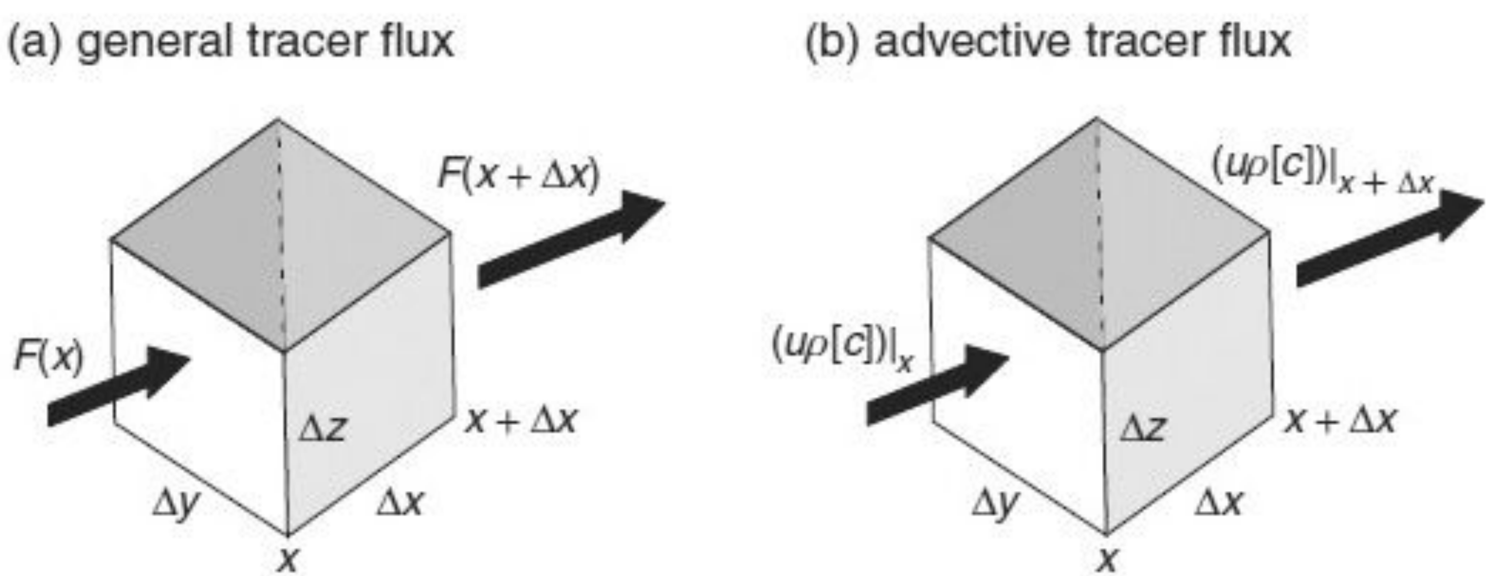
- Gulf Stream velocities are about 5 cm/sec at the bottom, up to more than 100 cm/sec at the top of the ocean. Assume an average of 20 cm/sec for this simple calculation.
- Assume a width of the current of 100 km
- Assume a depth of the current of 5 km
- The area of the G.S. is then 500 km^2
- Volume transport is then 20 cm/sec x 500 km^2



= 100 x 10^6 m^3/sec = 100 Sverdrups = 100 Sv

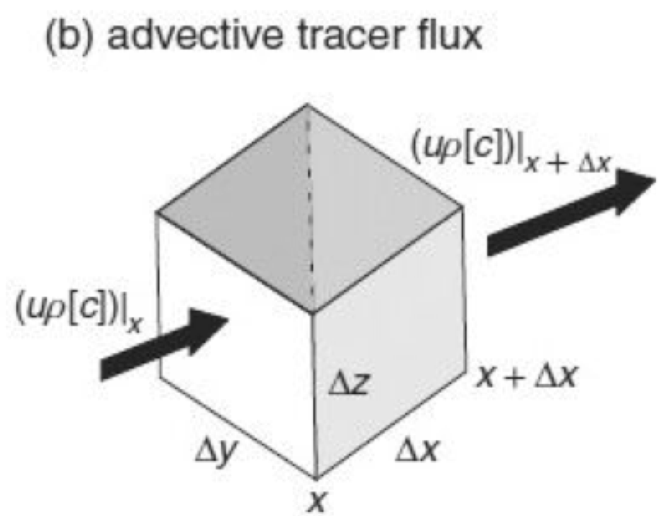
Flux

- A measure of transport
- Transport can be measured by the flux
  - Amount of mass/carbon/nutrients moving through a cross section per unit time

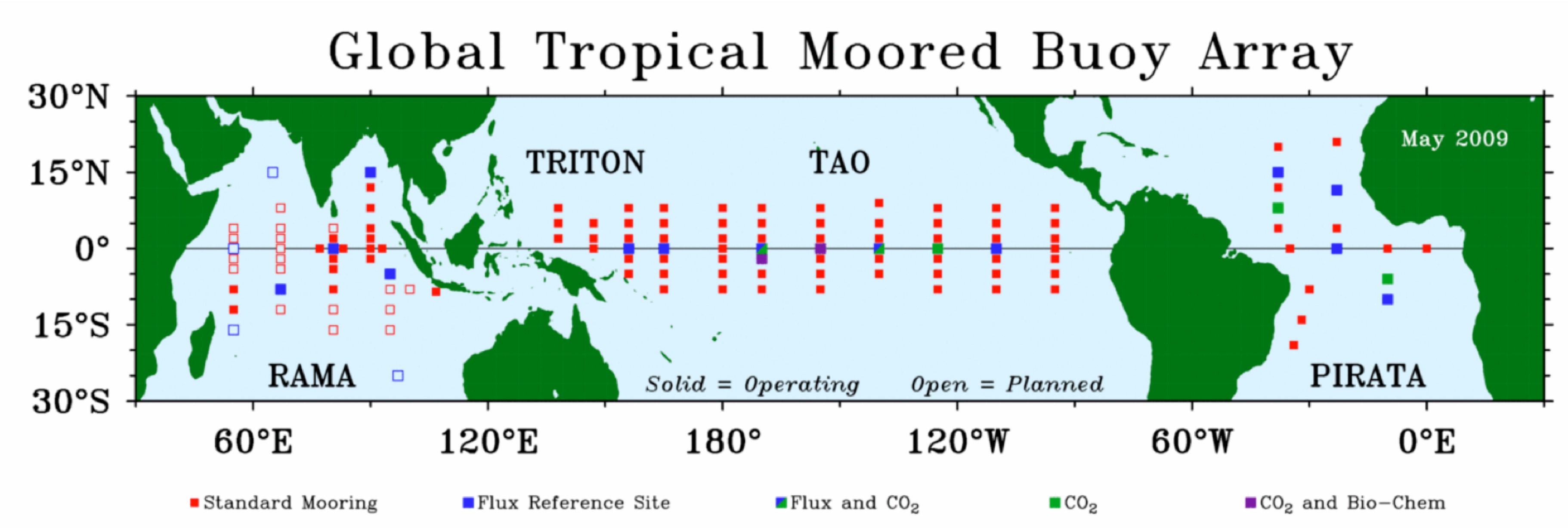


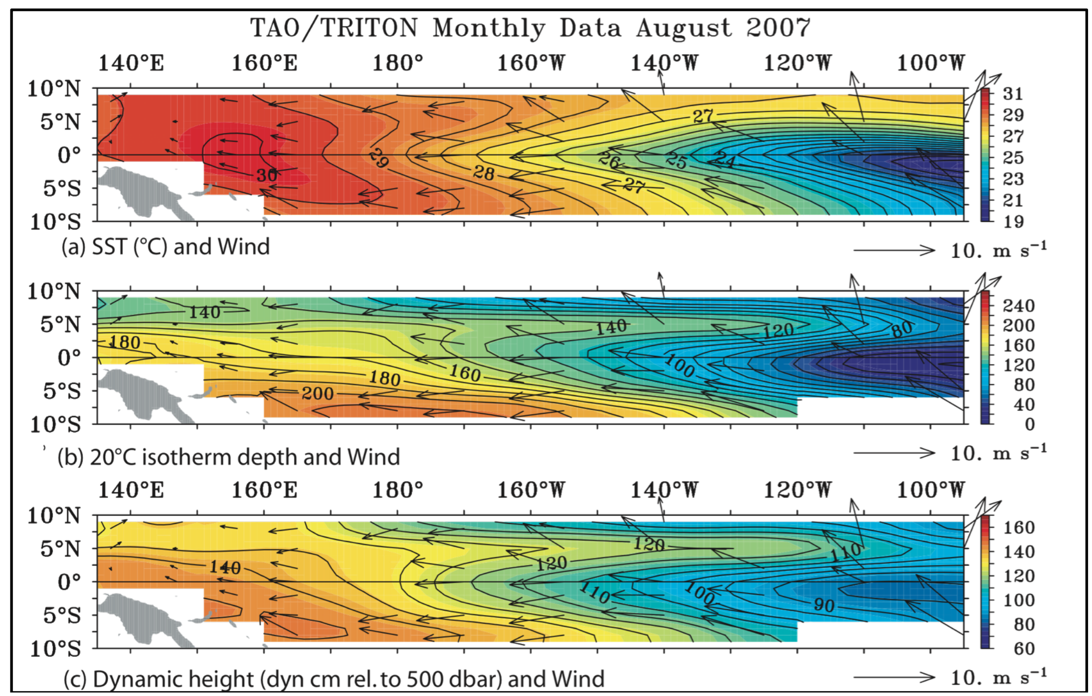
Advective tracer flux

- Tracer continuity equation
  - For a cross section of unit area:
    - Volume flux = (velocity)
    - Advective flux = (velocity) x (Concentration per vol)
      = (velocity) x (density) x (Conc. per mass)



TAO array: tropical ocean - atmosphere





Lagrangian kinematics

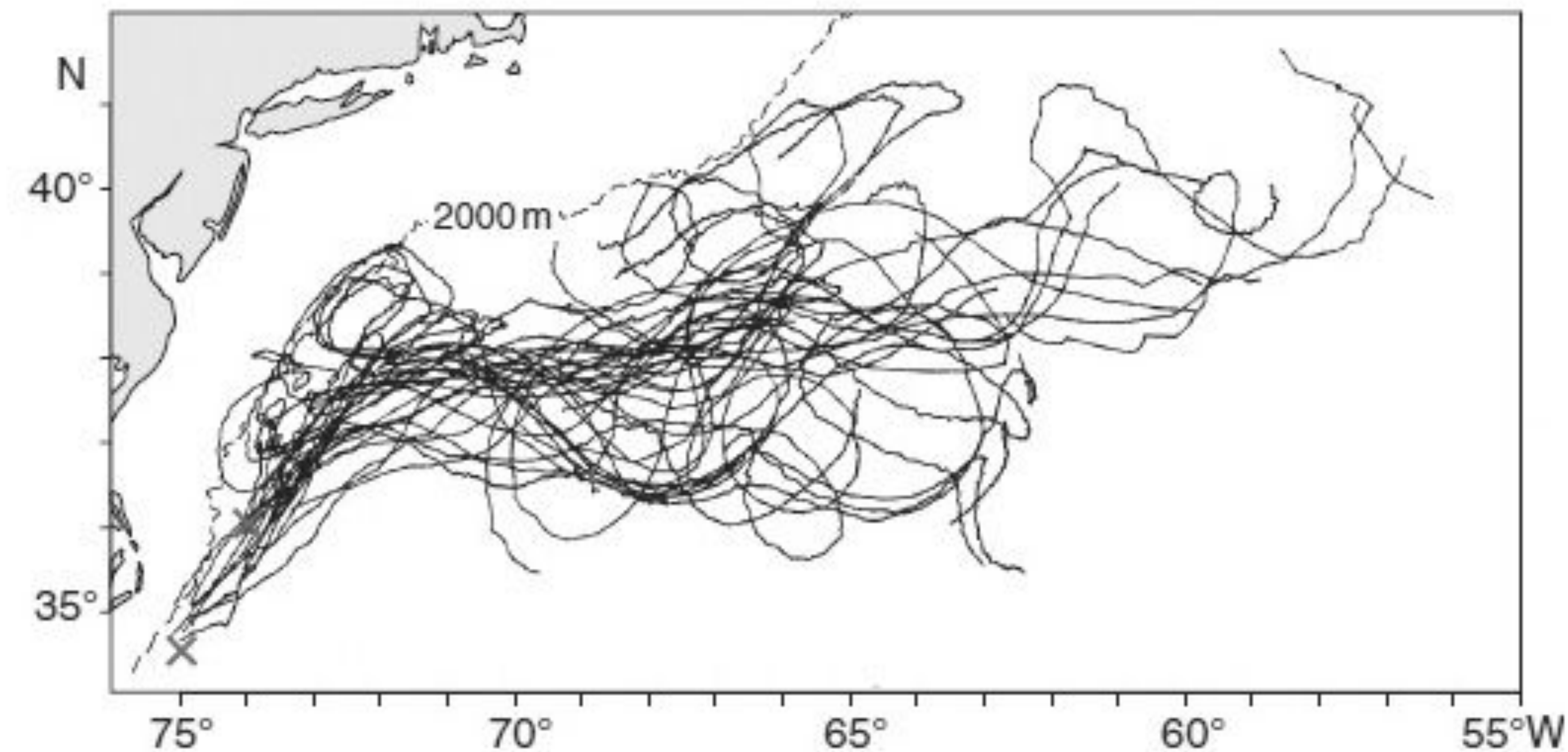
- Velocity = d/dt of the particle position
  - Consider following a drifter's trajectory

$$\frac{dx}{dt} = u, \quad \frac{dy}{dt} = v, \quad \frac{dz}{dt} = w$$

- Conservation laws

$$\frac{dc}{dt} = (\text{source of c})$$

Float trajectories



Eulerian kinematics

- Velocity field

$$u = u(x, y, z, t)$$
$$v = v(x, y, z, t)$$
$$w = w(x, y, z, t)$$

→ Consider data reported from a mooring

- Tracer field

$$T = T(x, y, z, t)$$
$$S = S(x, y, z, t)$$

- Conserved property: whiteboard discussion

$$\underbrace{\frac{dc}{dt}}_{Lagrangian} = 0 \qquad \underbrace{\left(\frac{\partial}{\partial t} + u \cdot \nabla\right) c = 0}_{Eulerian}$$

Expression of advection using vector calculus

- Nabla $$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

- Inner product with velocity

$$\mathbf{u} = u\hat{x} + v\hat{y} + w\hat{z}$$

$$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$$\mathbf{u}\cdot\nabla = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

Continuity equation

- (Rate of change) = (input) – (output)

  = (Flux convergence)

  – One dimensional example

$$\frac{\partial}{\partial t}(\rho C) = -\frac{\partial F}{\partial x}$$

  – Three dimensional version

$$\frac{\partial}{\partial t}(\rho C) = -\nabla \cdot \mathbf{F}$$

Mass continuity

- Conservation of mass expressed in Eularian frame
  - Rate of change following a moving water "parcel"

$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla\right) = \frac{D}{Dt}$$

$$\frac{D\rho}{Dt} + \rho \nabla \cdot \mathbf{u} = 0$$

  - In oceanography, we often use incompressible form

$$\nabla \cdot \mathbf{u} = 0$$

*If the divergence of volume flux is zero, the fluid is incompressible

Review quiz

- The water mass with the largest volume is:
  - (1) Pacific Deep Water
  - (2) Antarctic Intermediate Water
  - (3) Mode Water
  - (4) Labrador Sea Water

Review quiz

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Review quiz

- A Lagrangian measurement
  - (1) follows the water
  - (2) is made from a ship
  - (3) is made from a mooring
  - (4) is made using a satellite

Review quiz

- A Lagrangian measurement
  - (1) follows the water
  - (2) is made from a ship
  - (3) is made from a mooring
  - (4) is made using a satellite

Review quiz

- A water parcel has a temperature of 2 degree C at the surface. If the water parcel is adiabatically moved down to 2000 dbar, what would be its temperature?
  - (1) Lower than 2 degree
  - (2) At 2 degree
  - (3) Higher than 2 degree
  - (4) Not enough information is given to answer this

Review quiz

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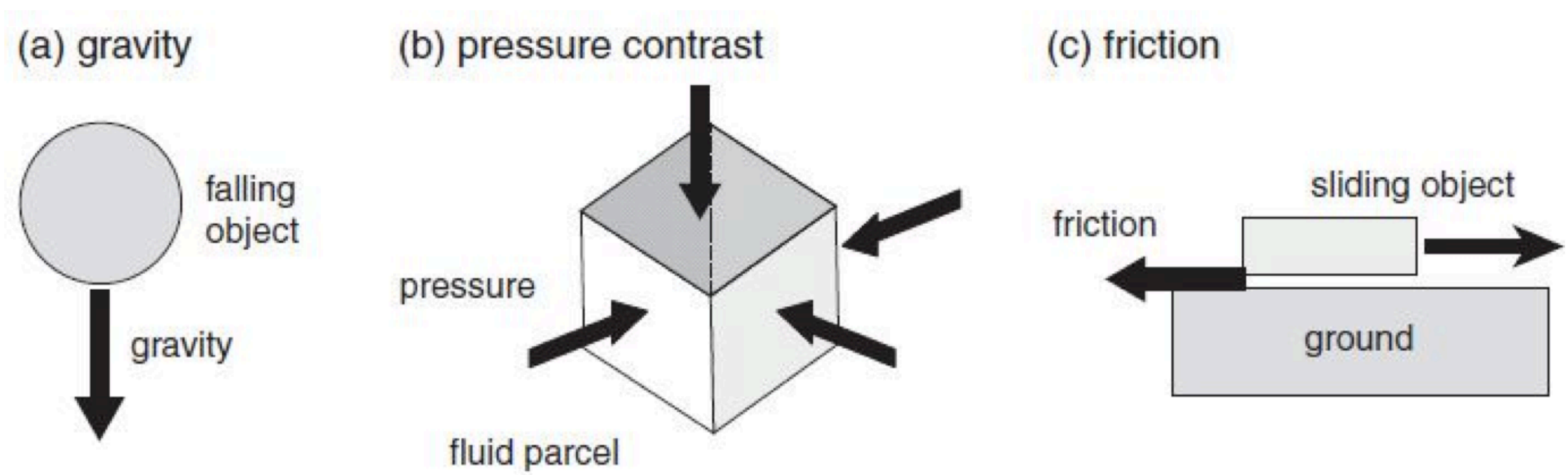
Newton's law

- m**a** = **F** applied to a Lagrangian water parcel (with a unit volume)

$$\mathbf{a} \to \frac{D\mathbf{u}}{Dt}$$ Acceleration : rate of change in velocity

$$m \to \rho$$ density : mass per unit volume

Three types of forces we consider here:



Gravitational force

- Points to the center of the Earth
  - In Cartesian coordinate (x, y, z), it is vertically downward
  - Unit vector in (x, y, z) direction is (**i**, **j**, **k**).

$$F_{gravity} = -\rho g\mathbf{k}$$

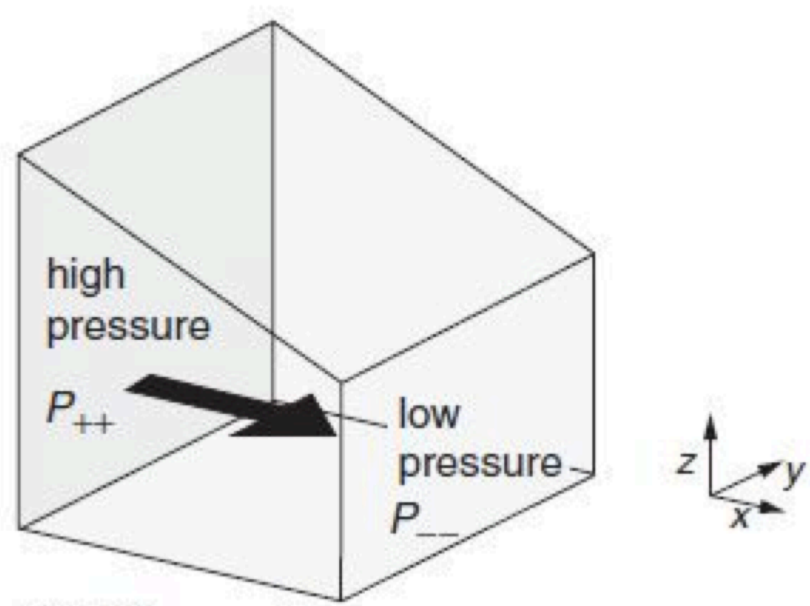
Pressure Gradient Force

- Pressure variation exerts force on the fluid parcel
- In this example, pressure decreases in x direction

$$\frac{\partial P}{\partial x} < 0$$

$$F_{PGF} = -\frac{\partial P}{\partial x}\mathbf{i}$$

$$F_{PGF} = -\nabla P$$



Newton's law with gravity and PGF

- m**a** = **F** applied to a Lagrangian water parcel (with a unit volume)

$$\mathbf{a} \rightarrow \frac{D\mathbf{u}}{Dt}$$

$$m \rightarrow \rho$$

$$\rho \frac{D\mathbf{u}}{Dt} = -\rho g \mathbf{k} - \nabla P + \mathbf{F}$$

In x-, y-, z- components

$$\rho \frac{Du}{Dt} = -\frac{\partial P}{\partial x} + F_x$$

$$\rho \frac{Dv}{Dt} = -\frac{\partial P}{\partial y} + F_y$$

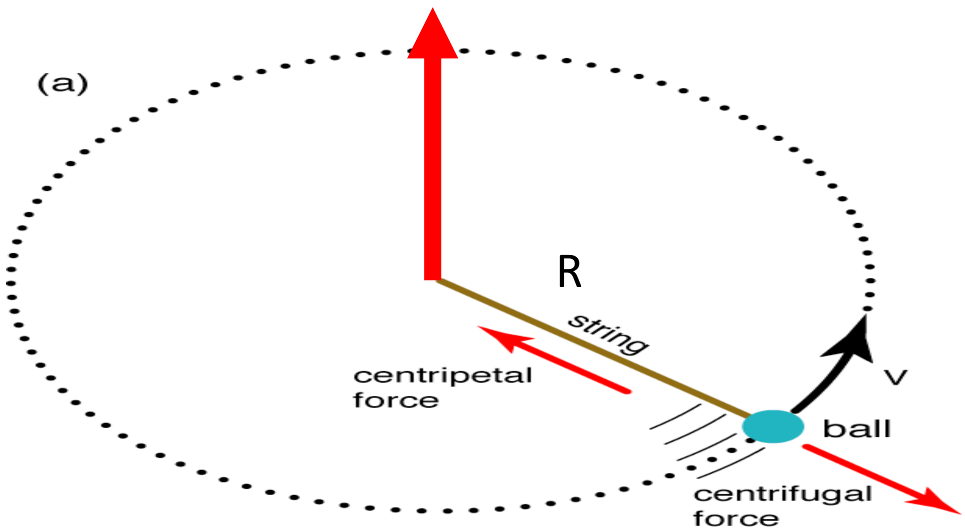
$$\rho \frac{Dw}{Dt} = -\rho g - \frac{\partial P}{\partial z} + F_z$$

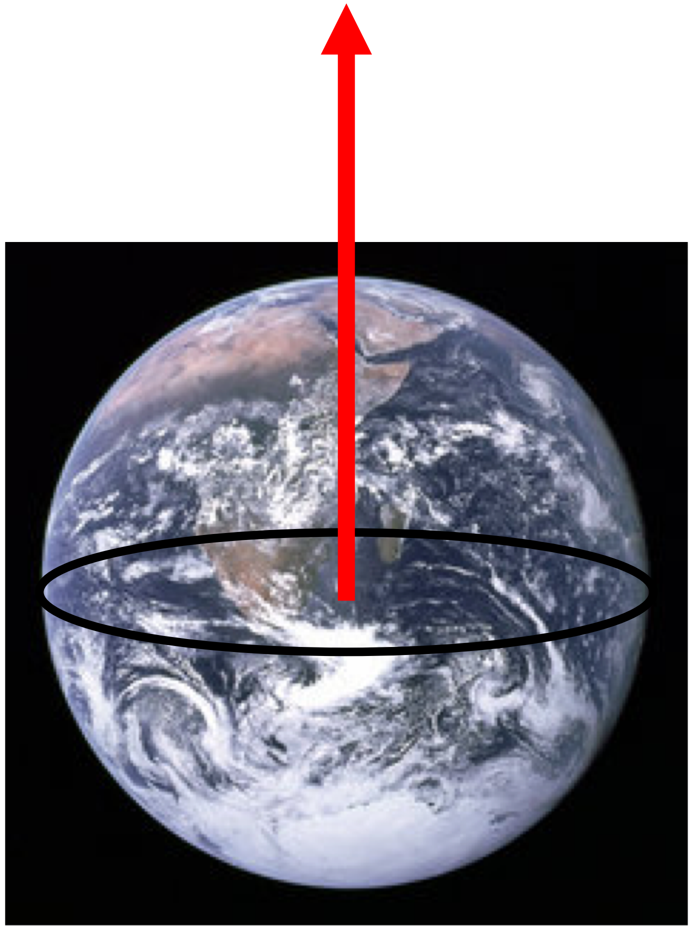
Primitive equation in inertial frame of reference (non-rotating).

Rotating coordinates

- **Rotation vector** that expresses direction of rotation and how fast it is rotating:

  vector pointing in direction of thumb using right-hand rule, curling fingers in direction of rotation





Rotating coordinates

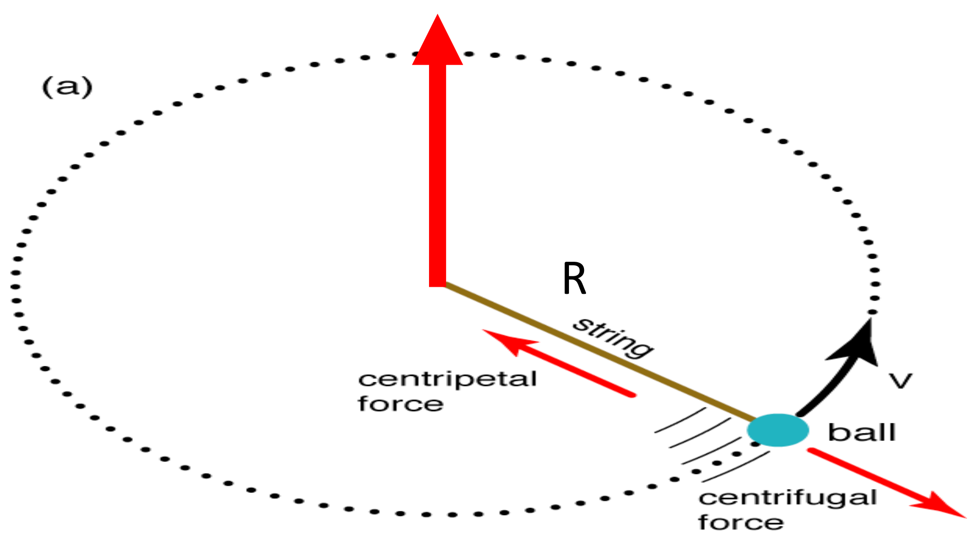
- The Earth is rotating. We measure things relative to this **"rotating reference frame".**
- Quantity that tells how fast something is rotating:

  Angular speed or angular velocity **Ω = angle/second**

  360° is the whole circle, but express angle in radians (2π radians = 360°)

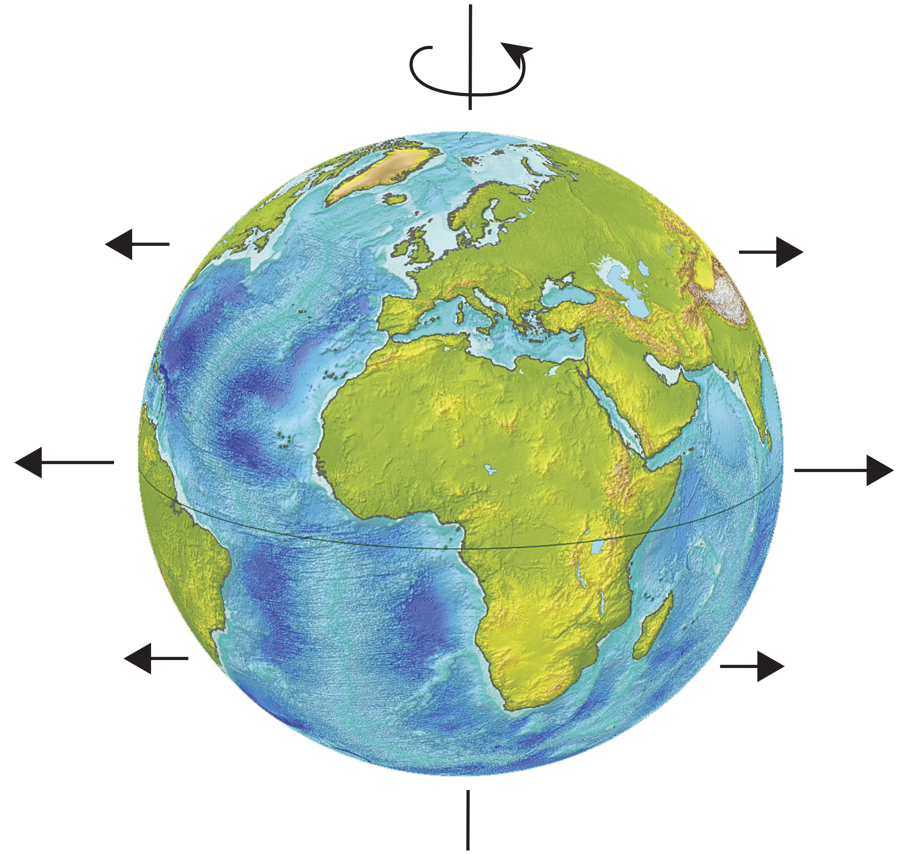
  For Earth: 2π / 1 day = 2π / 86,400 sec = 0.707×10^{-4} /sec

  Also can show $\Omega = v/R$ where v is the measured velocity and R is the radius to the axis of rotation (therefore $v = \Omega R$)



The effect of rotation

- Additional forces due to the Earth's rotation
  - **Centrifugal force**
    - Direction: Radially outward w.r.t. the axis of rotation
    - Magnitude: Proportional to $(\text{rotation rate})^2$ and distance from the axis of rotation
    - Effect: Modifies gravity



Effect of centrifugal force on the Earth

Centrifugal force acts on the ocean and earth. It is pointed outward away from the rotation axis.

Therefore it is maximum at the equator (maximum radius from axis) and minimum at the poles (0 radius).

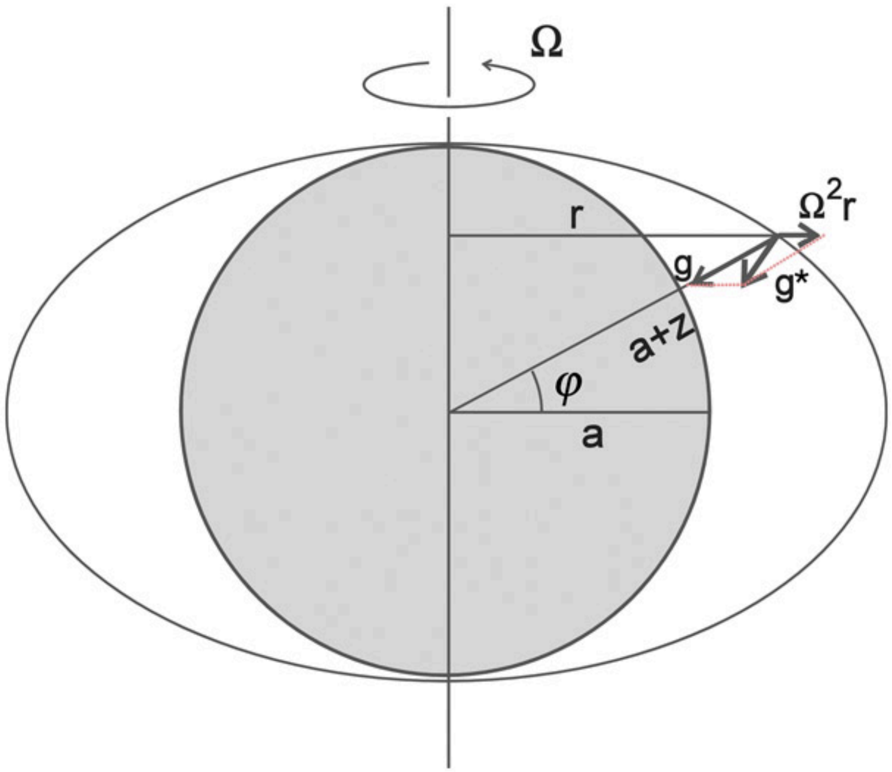
$\Omega = 0.707 \times 10^{-4}$ /sec

At the equator, R ~ 6380 km so **$\Omega^2 R = .032$ m/s^2**

Compare with gravity = 9.8 m/s^2

Geoid on a spherical planet

- Centrifugal force bulges out the surface away from the axis of rotation
- Geoid ≈ ellipsoid
- Apparent gravity is deflected outward
- *The Earth's geoid has irregularities due to spatially varying gravity (shape) of the planet*



More on the geoid

When we calculate horizontal pressure gradients, or height of sea surface, they are relative to a “reference level” along which the pressure gradient vanishes (no force, no motion)

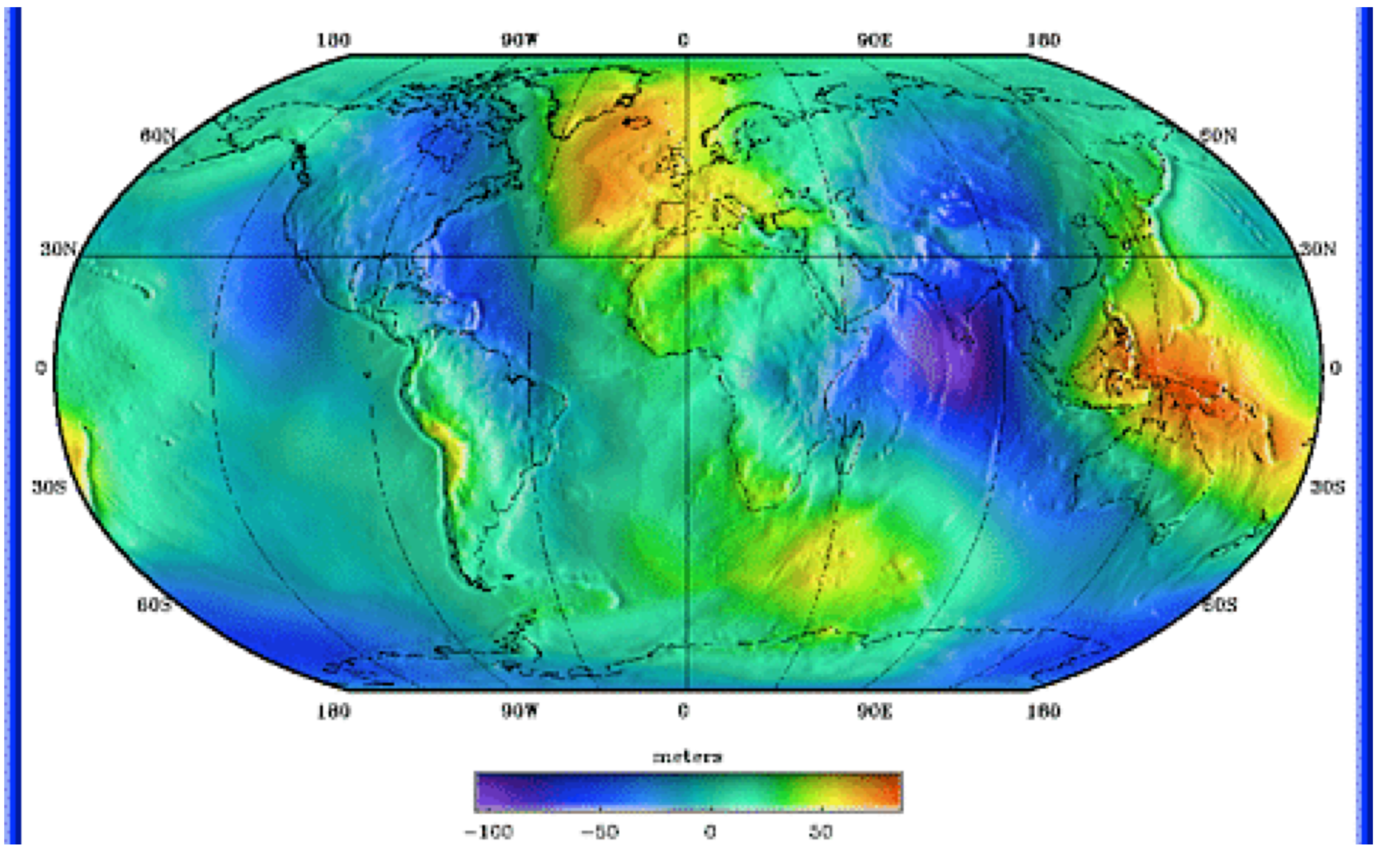
What do we mean by mean sea level?

Earth’s mass is not distributed evenly AND Earth is not a perfect ellipsoid. That is, the “reference level” surface is not locally “flat”.

This “reference surface”, surface of constant geopotential, i.e. along which the gravitational force has no changes, is the “geoid”

Observed geoid

- Earth's mass is not distributed evenly AND Earth is not a perfect ellipsoid



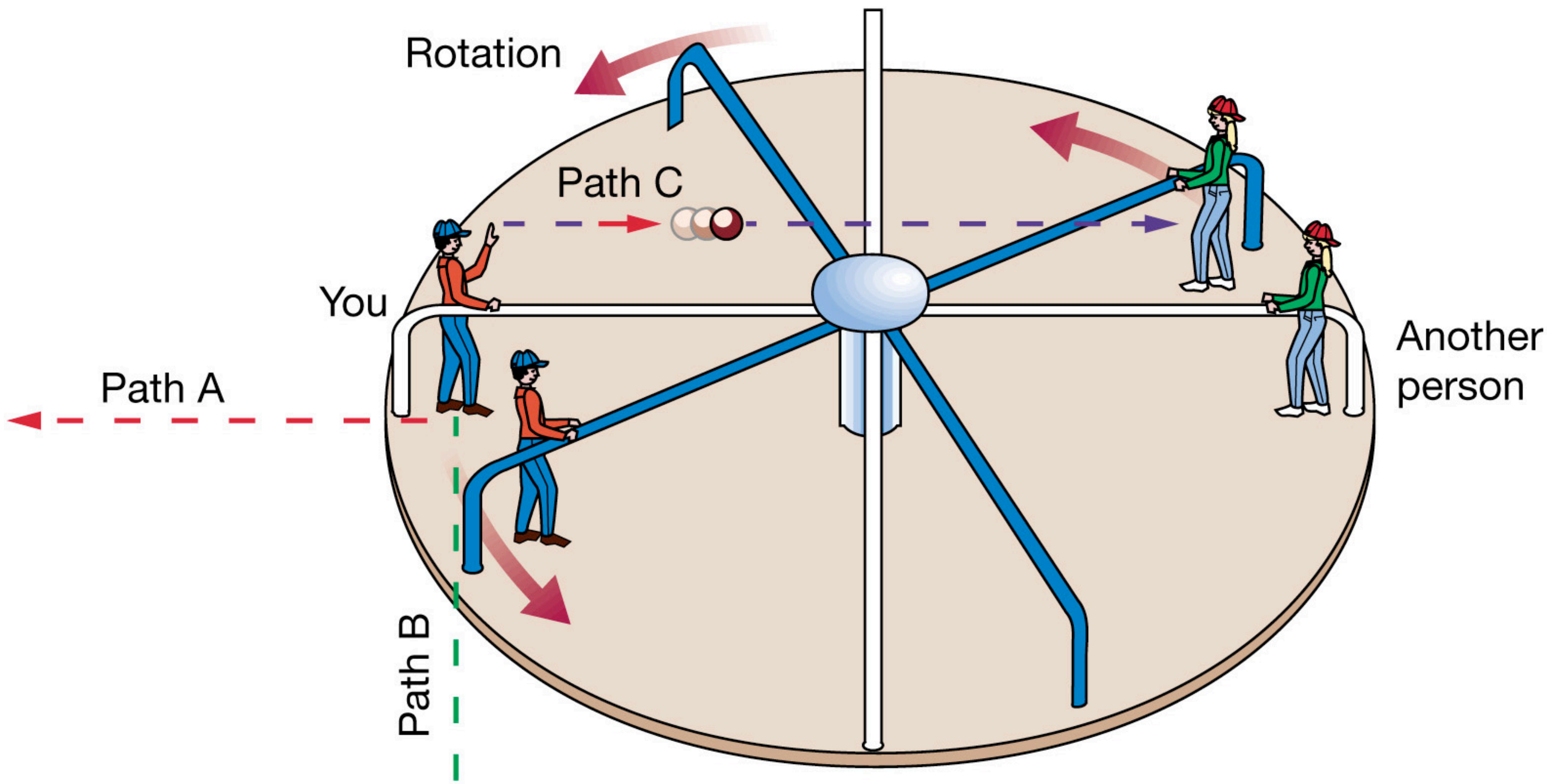
Geoid map using **EGM96 d**ata, from http://cddis.gsfc.nasa.gov/926/egm96/egm96.html

The effect of rotation

- Forces due to the Earth's rotation
  - Centrifugal force
  - **Coriolis force**
    - Direction: Perpendicular to the motion of the object
    - Magnitude: Proportional to the speed of the object and the rotation rate
    - Effect: **Deflects moving object**

Coriolis force

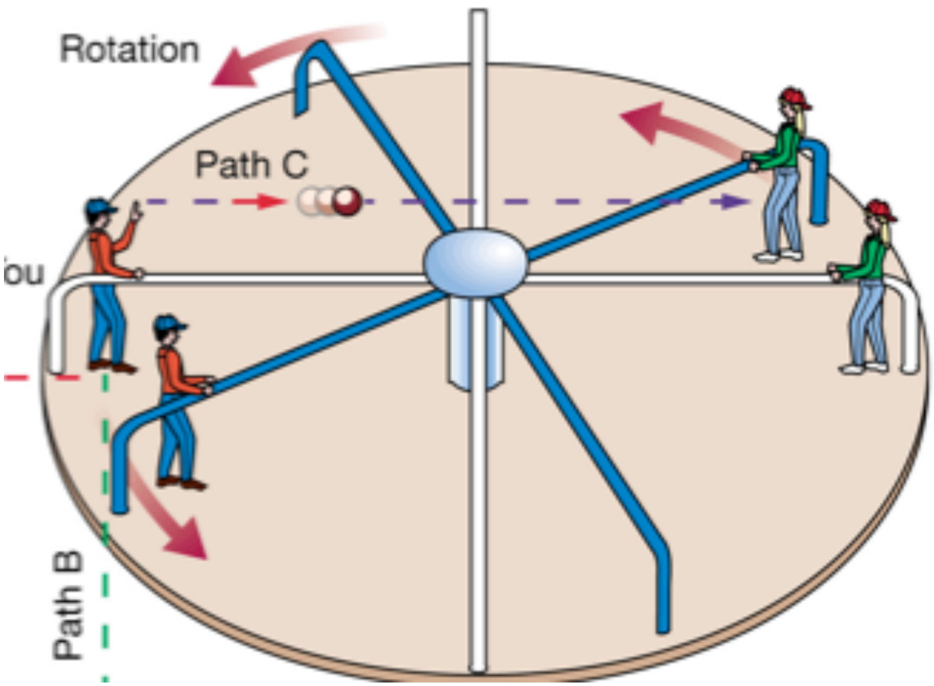
Exmaple: playing catch on a merry-go-round



- Straight path in inertial (non-rotating) frame
- Deflection to the right in rotating frame

Coriolis force

- In northern hemisphere, planetary rotation is **counter-clockwise**.
- A moving object is **deflected to the right** in rotating frame of reference



Rotating table experiments

- Deflection of marbles
- IF the table is rotating counterclockwise, the marble always deflect to the right of the direction of its motion
- IF the table is rotating clockwise, the marble always deflect to the left of the direction of its motion

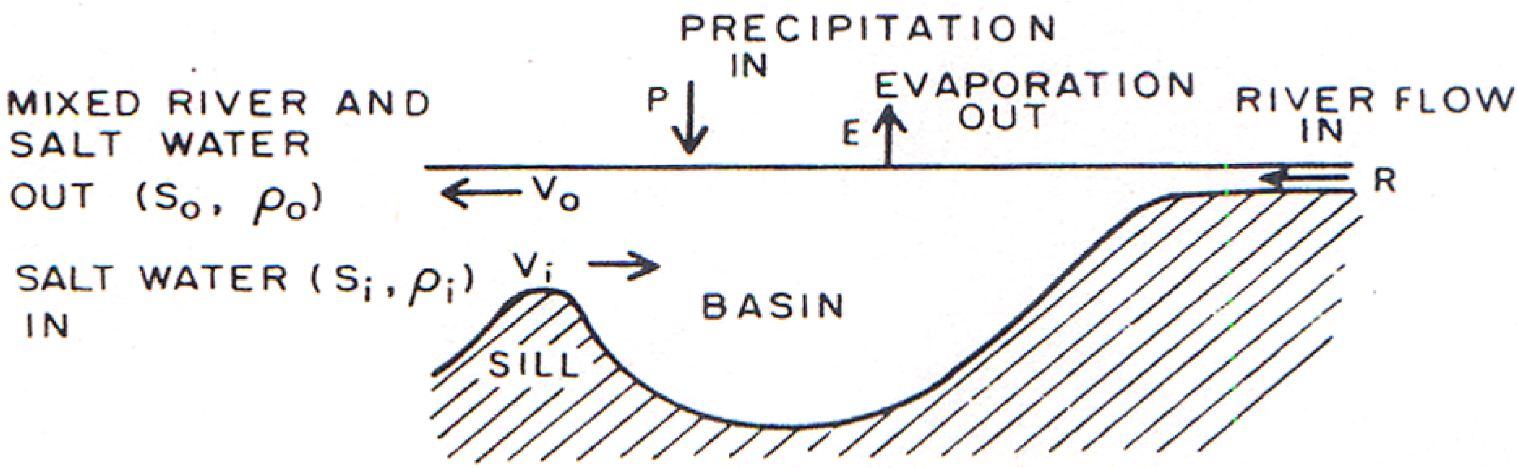
CONTINUE TO NEXT WEEK

Conservation of mass in a semi-enclosed basin

(1) Mass conservation:

$F \equiv \rho_o V_o - \rho_i V_i =$ $(R + AP) - AE$

(F = "freshwater" = net amount of rain in kg, evaporation, runoff into the area); A = surface area in m^2; R = runoff in kg; P = precipitation kg/m^2; E = evaporation per unit area kg/m^2

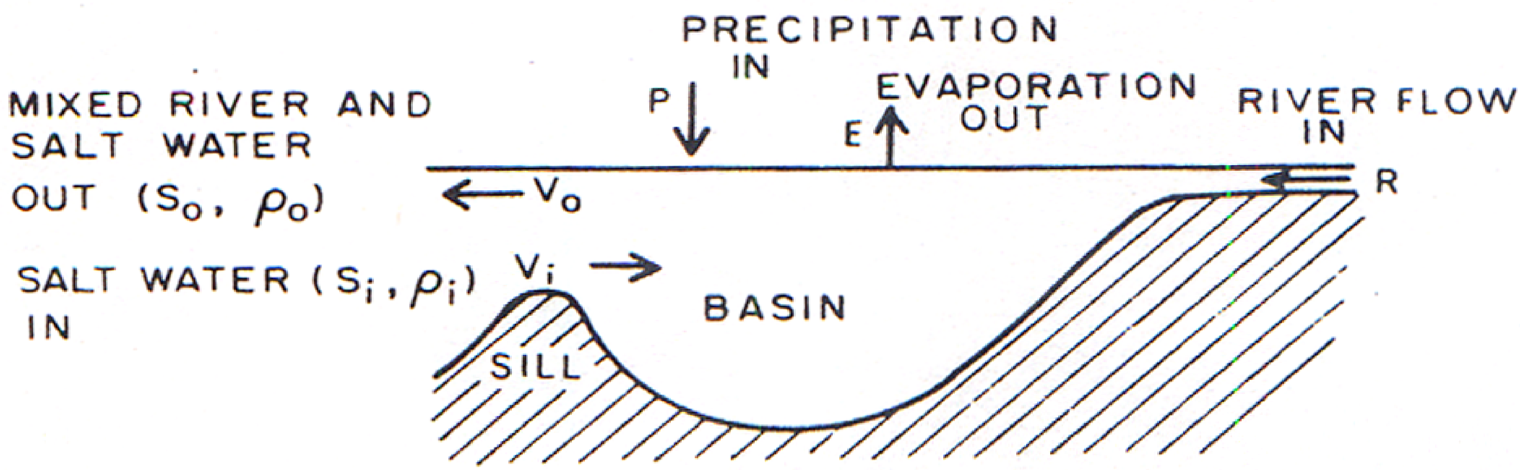


DPO Fig. 5.1

Conservation of salt

(2) Salt conservation: $V_i \rho_i S_i = V_o \rho_o S_o$

*Note that river inflow, evap and precip do not carry salt



DPO Fig. 5.1

Conservation of freshwater

Mass: F $= \rho_o V_o - \rho_i V_i = (R + AP) - AE$

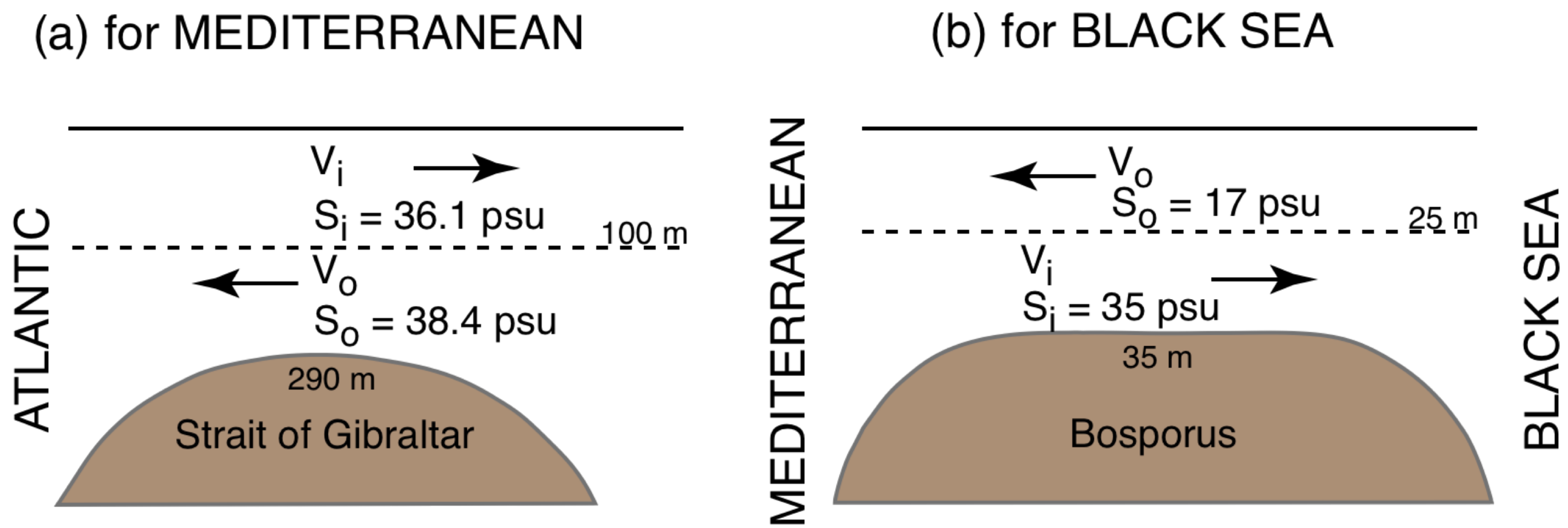
Salt: $V_o \rho_o S_o - V_i \rho_i S_i = 0$

Eliminate $V_i \rho_i$ and write the relationship between F and salinity.

$F \sim \rho V (S_i - S_o) / S_i$

So the freshwater input calculated from the difference in salinity between inflow and outflow equals the net precipitation, evaporation, runoff

Freshwater transport: Mediterranean and Black Seas



Evaporative basin

DPO Figure 5.3

Runoff/precipitation