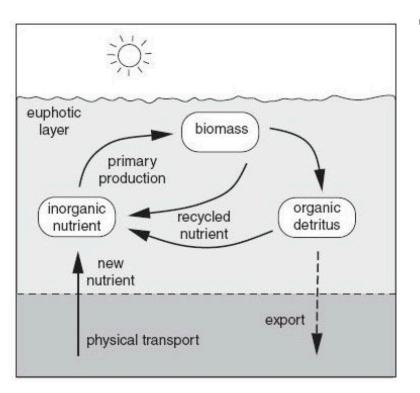
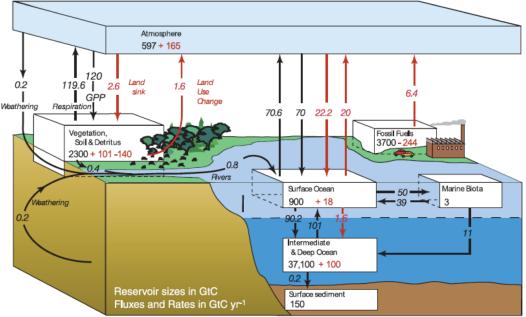
Week 12: Nutrient and carbon cycling

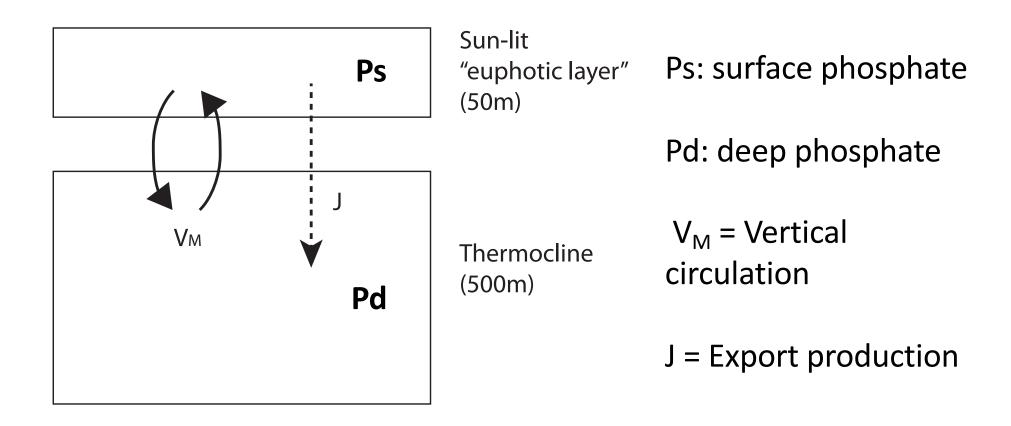
1 GtC (giga-tons C) = 1 PgC (peta-grams C) = 10¹⁵ gC Figure credit: IPCC, N. Gruber





10-15PgC/yr: NCP ~ export of organic carbon from surface euphotic layer to the thermocline and deep waters

A simple nutrient cycle model



Downward organic flux (J) is balanced by the vertical transport at steady state: the balance regulates the vertical gradient

2-box nutrient cycle model

$$V_s \frac{dP_s}{dt} = V_M (P_d - P_s) - \lambda_{bio} V_s P_s$$

$$V_d \frac{dP_d}{dt} = V_M (P_s - P_d) + \lambda_{bio} V_s P_s$$

 $V_{S} : volume of the surface box (m^{3})$ $V_{d} : volume of the deep box (m^{3})$ $V_{M}: vertical circulation rate (m^{3}/s)$ $P_{S}: surface P concentration (molP/m^{3})$ $P_{D}: deep P concentration (molP/m^{3})$ $\lambda_{bio}: biological P export rate (1/s)$

Steady state solutions

Conservation of nutrient

$$V_s P_s + V_d P_d = const$$

Solution for surface nutrient

$$P_s = \frac{P_d}{1 + \frac{\lambda_{bio}V_s}{V_M}}$$

Solution for export production

$$J = \left(\frac{\gamma}{1+\gamma}\right) V_M P_d$$

Some numbers...

- Each year, human activity emits about 10PgC
 - About half of it remains in the atmosphere
 - Remaining half goes into the land and the ocean
- Each year, marine photosynthesis fixes 50PgC of carbon dioxide into organic matter

About 30% of it sinks to the thermocline/deep ocean

- The oceans contain 90+% of carbon molecule in the atmos/land/ocean system
 - 38,000 PgC as dissolved inorganic carbon

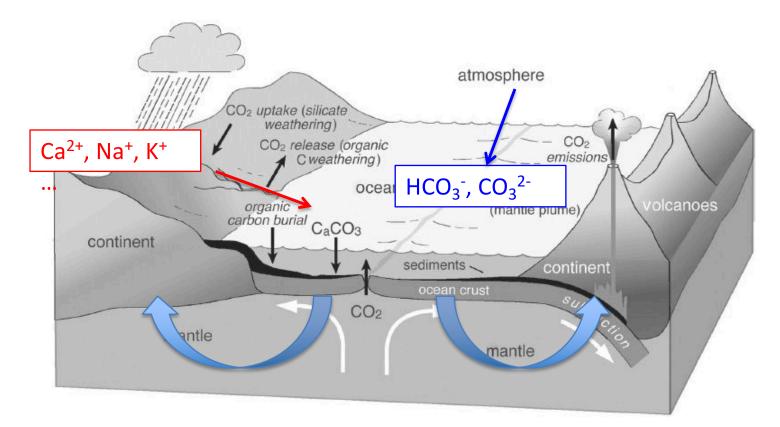
Why the oceans contain such a large amount of carbon?

CO₂ is soluble and reacts with water
 DIC (dissolved inorganic carbon) or Total Carbon

$$DIC = [\underbrace{CO_2^*}_{<1\%}] + [\underbrace{HCO_3^-}_{\sim 90\%}] + [\underbrace{CO_3^{2-}}_{\sim 10\%}]$$

– Why most of carbon molecules are in bicarbonate and carbonate ions?

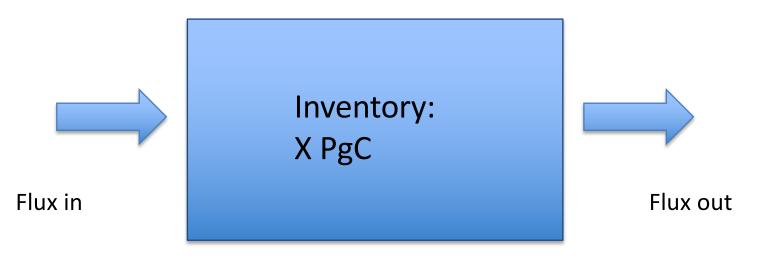
Geological carbon cycle



- Cycling of carbon between rocks, air and seawater
- "Carbonate Buffer System": positive and negative ions must balance each other → electron balance

Residence time of nutrients and carbon

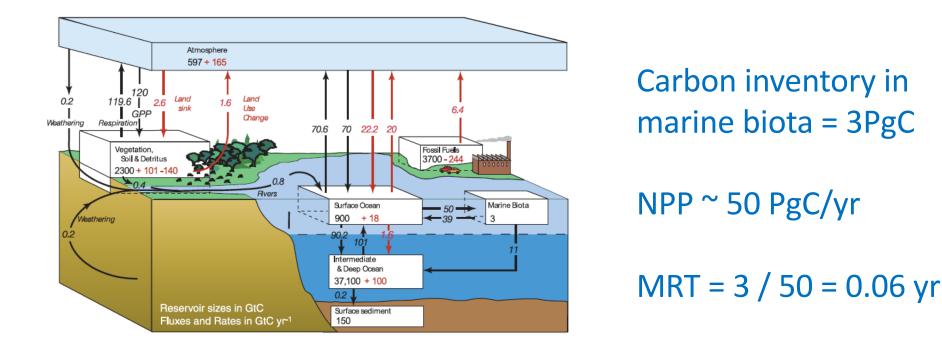
MRT = (mean residence time)
 = (inventory) / (flux)



Fluxes: Y PgC/yr

MRT = (X, PgC)/(Y, PgC/yr) = X/Y, yr

Example: what is the MRT of carbon biomass in the marine/land biota?



c.f. Carbon inventory in land biota = 2300 PgC NPP ~ 50 PgC/yr MRT = 2300 / 50 = 46 yr

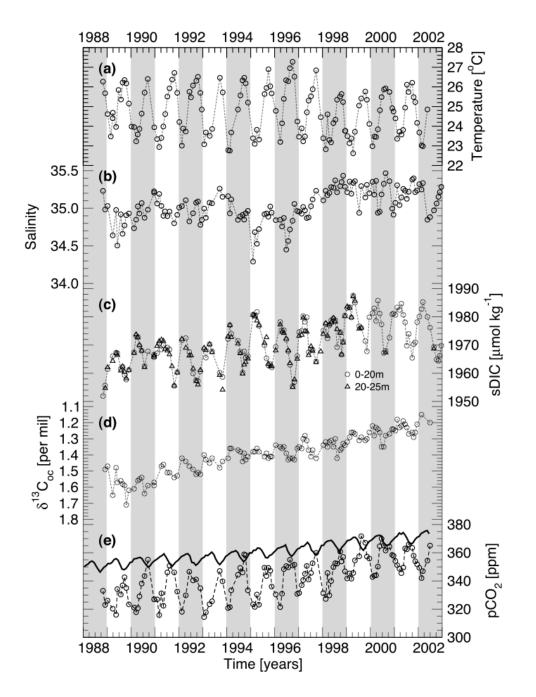
Observed seasonal cycle in Hawaii

• Brix et al. (2004)

During the summer, low DIC coincides with high SST → Biological C uptake and export.

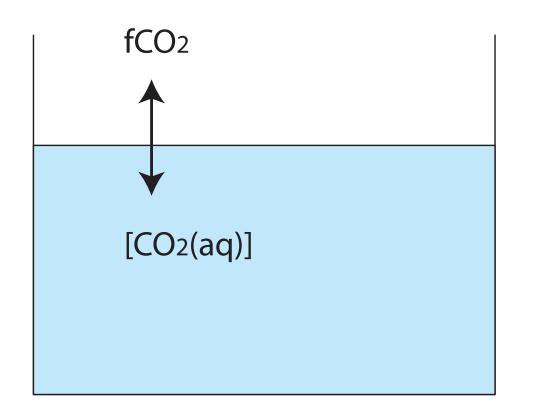
pCO2 is high during the summer (low solubility)

Long term increase in C and decrease in C-13 (Suess effect)



Solubility of gases in the seawater

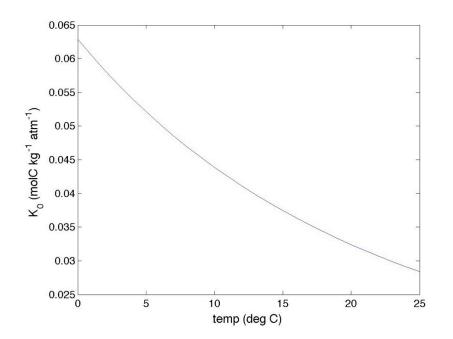
- CO₂ is a soluble gas
- Solubility of CO2 = how many CO2 molecule can be dissolved in a unit mass of seawater at equilibrium



Note: Surface ocean is not always at equilibrium... sometimes under-saturated and super-saturated, depending on many factors including biological C uptake, heating/cooling, transport, etc.

Solubility: Henry's law

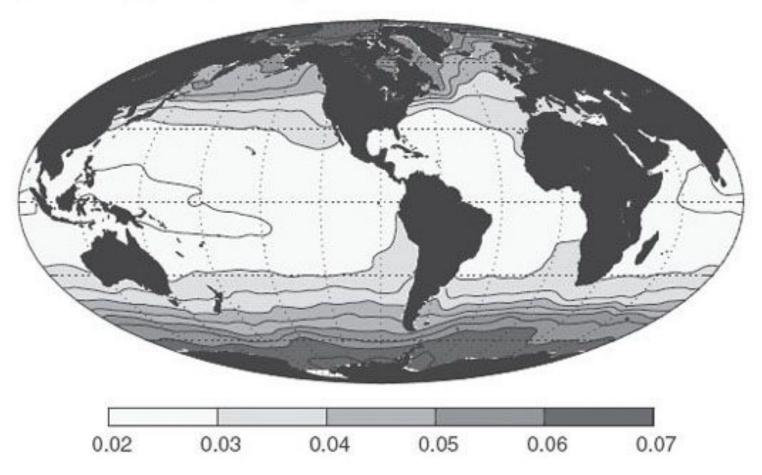
- CO₂ solubility (here, K₀) is a function of temperature and salinity (units: molC m⁻³ atm⁻¹)
- Square bracket [] indicates concentration
- Fugacity (fCO2) and partial pressure (pCO2) are almost equivalent (~1%) difference → activity vs concentration



 $[CO_2] = K_0 fCO_2$ $K_0 = K_0(T, S)$ $fCO_2 \sim pCO_2$

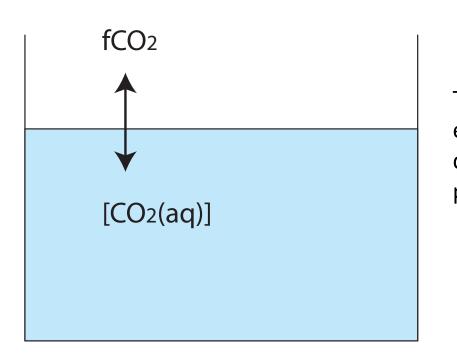
Mapping the solubility of CO₂

(a) solubility (mol kg⁻¹ atm⁻¹)



Air-sea exchange

- Surface ocean exchanges gases with the overlying atmosphere: how does it happen?
- Two types of gas exchange kinetics
 - Diffusive gas exchange
 - Bubble mediated gas exchange

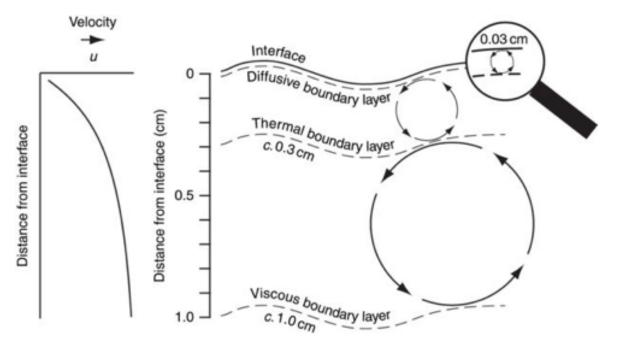


The rate at which the gas molecule is exchanged at the air-sea interface depends on the types of physical processes responsible.

Diffusive gas exchange

- Molecular diffusion at the diffusive boundary layer
 - Enhanced by the turbulent mixing
- G: gas transfer coefficient (m s⁻¹)

$$(outflux) = G([CO_2] - [CO_{2,sat}])$$
$$= GK_0(pCO_{2,ocn} - pCO_{2,atm})$$



Note: diffusive flux is down-gradient between ocean and atmosphere.

The rate coefficient (G) depends on the nearsurface turbulence

Controls on air-sea gas transfer rates

- G: gas transfer coefficient (ms⁻¹)
 - Turbulence in the surface ocean
 - Depends on:
 - Surface wind speed
 - Schmidt number (Sc)
 - Viscosity of the water

$$S_C = rac{Viscosity}{\text{Diffusivity}}$$

- Molecular diffusivity of gas in the water

$$G = G_0(U_{spd})S_C^{-n}$$

Wind dependence of G

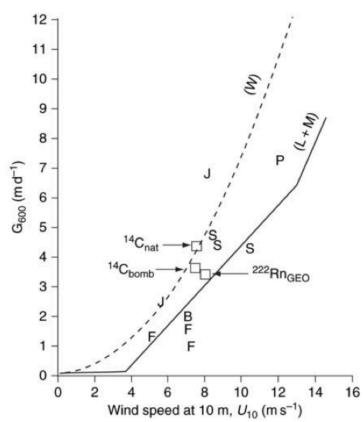


Figure 10.6. Gas transfer rates normalized to a Schmidt number of 600, G_{600} , from global ¹⁴C and localized ²²²Rn measurements in the ocean as a function of wind speed measured at 10 m above the air–water interface, U_{10} . ¹⁴C_{nat} and ¹⁴C_{bomb} are the gas exchange rates determined from natural and bomb-produced ¹⁴C discussed in the text. The box labeled Rn_{GEO} represents the average result from over 100 ²²²Rn measurements in the oceans during the GEOSECS

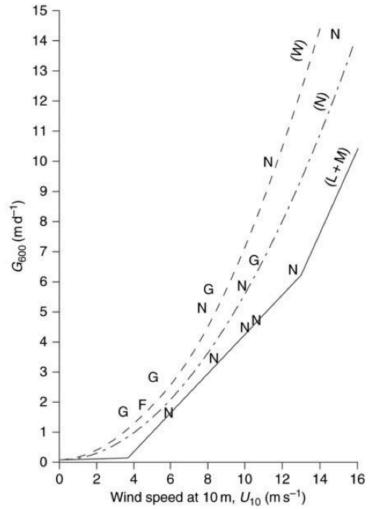
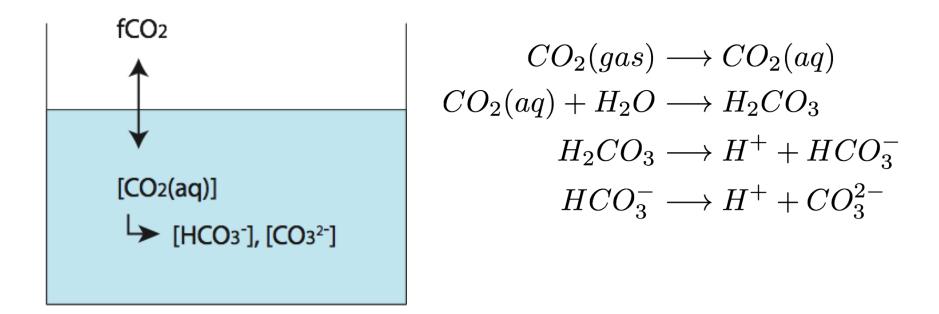


Figure 10.7. Gas transfer velocity, G_{600} , as a function of wind speed, U_{10} , for all dual tracer experiments. N is from the North Sea,

Carbonate chemistry

• Formation of HCO3 and CO3 ions



Dissolved inorganic carbon

- CO₂ reacts with water to form bicarbonate and carbonate ions
- In aquatic system, carbon inventory is expressed as the sum of inorganic C species

$$[CO_{2}*] = [CO_{2}(aq)] + [H_{2}CO_{3}]$$
$$DIC = [CO_{2}*] + [HCO_{3}^{-}] + [CO_{3}^{2-}]$$
$$_{\sim 1\%}^{\sim 90\%} \qquad _{\sim 10\%}^{\sim 10\%}$$

Local Chemical Equilibrium

These reactions reaches chemical equilibrium within a few minutes

$$CO_2 + H_2O \longleftrightarrow H^+ + HCO_3^-$$
$$K_1(T, S, p) = \frac{[H^+][HCO_3^-]}{[CO_2^*]}$$

$$HCO_{3}^{-} \longleftrightarrow H^{+} + CO_{3}^{2-}$$

 $K_{2}(T, S, p) = \frac{[H^{+}][CO_{3}^{2-}]}{[HCO_{3}^{-}]}$

Partitioning of carbon molecule

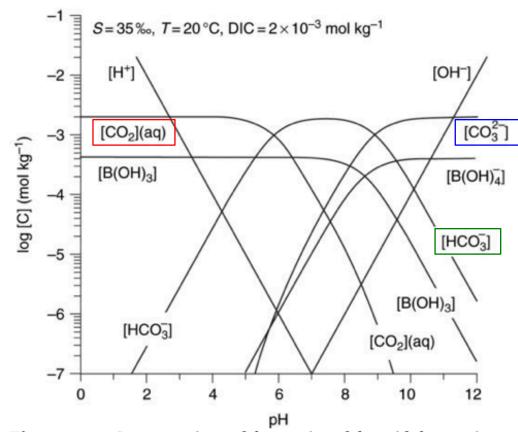


Figure 4.2. Concentrations of the species of the acid–base pairs of carbonate, borate and water in seawater as a function of pH. (Salinity, S = 35, temperature, T = 20 °C and DIC = 2.0×10^{-3} mol kg⁻¹.)

Partitioning of carbon molecule

$$DIC = [CO_2*] \left(1 + \frac{K_1}{[H^+]} + \frac{K_1K_2}{[H^+]^2} \right)$$
$$pH = -log_{10}[H^+]$$
$$pK_1 = -log_{10}K_1$$
$$pK_1 \sim 6, \ pK_2 \sim 9$$

At present climate, pH = 8,

$$\frac{K_1}{[H^+]} \sim 10^2, \ \frac{K_1 K_2}{[H^+]^2} \sim 10$$

Carbonate buffer

- Acid-neutralizing ability of the seawater
 - Initially adding CO₂ molecule increases [H⁺]
 - Additional [H⁺] reacts with carbonate ion
 - Bicarbonate increases twice the rate of initial CO2 addition
 - The change in [H⁺] is suppressed as a net effect

 $CO_2 + H_2O \longleftrightarrow H^+ + HCO_3^ H^+ + CO_3^{2-} \longleftrightarrow HCO_3^-$

$$CO_2 + CO_3^{2-} + H_2O \longleftrightarrow 2HCO_3^{-}$$

Alkalinity and pH of the seawater

- Consider electric neutrality
 - Cations (Na⁺, Mg²⁺, K⁺, Ca²⁺, …)
 - Anions (Cl⁻, HCO³⁻, CO^{3²⁻}, ...)

- Group cations and anions into strong and weak ions

$$[Na^+] + 2[Mg^{2+}] + [K^+] + 2[Ca^{2+}] \dots = [Cl^-] + [HCO_3^-] + 2[CO_3^{2-}] \dots$$

$$A_T = [Na^+] + 2[Mg^{2+}] + [K^+] + 2[Ca^{2+}] - [Cl^-]...$$
$$= [HCO_3^-] + 2[CO_3^{2-}]...$$

Carbonate alkalinity

- Alkalinity (A_T), carbonate alkalinity (A_c) and DIC
 - Carbonate alkalinity (A_c: about 90% of A_T)
 contains only bicarbonate and carbonate ions.

$$0.9A_T \sim A_C = [HCO_3^{-}] + 2[CO_3^{2-}]$$
$$DIC \sim [HCO_3^{-}] + [CO_3^{2-}]$$
$$A_C \sim [HCO_3^{-}] + 2[CO_3^{2-}]$$

Useful approximations

$$DIC \sim [HCO_3^-] + [CO_3^{2-}]$$

 $A_C \sim [HCO_3^-] + 2[CO_3^{2-}]$

Thus

$$[CO_3^{2-}] \sim A_C - DIC$$
$$[HCO_3^{-}] \sim 2DIC - A_C$$

Note: CO_2 addition will increase DIC but not alkalinity. HCO_3^- increases twice the rate of DIC at the expense of CO_3^{2-}

And,

$$[H^+] \sim K_2 \left(\frac{2DIC - A_C}{A_C - DIC} \right) \quad [CO_2*] \sim \frac{K_2}{K_1} \left(\frac{\{2DIC - A_C\}^2}{A_C - DIC} \right)$$

Control of pH

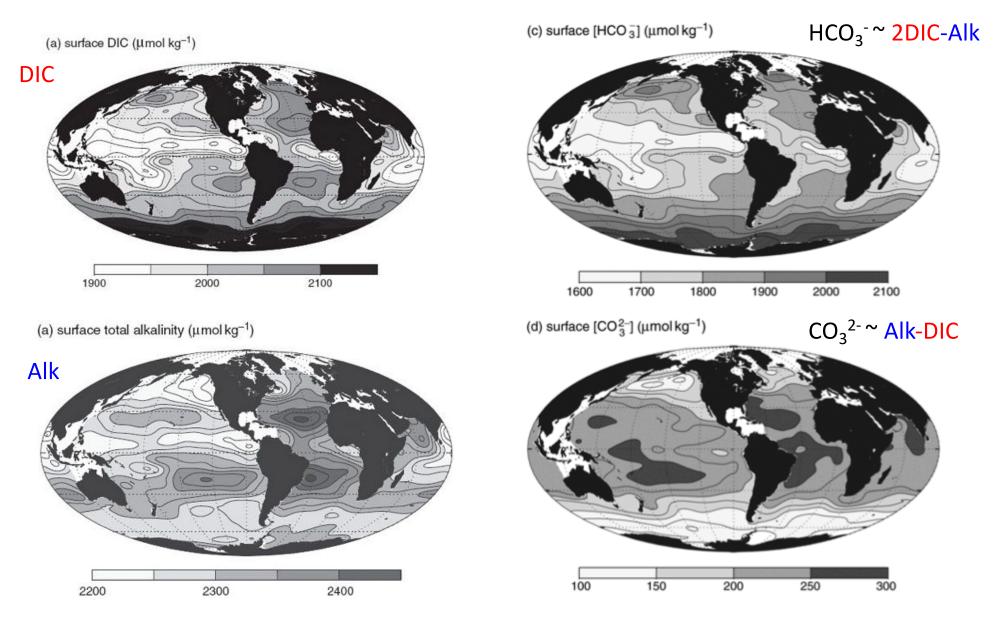
- On average the pH of seawater is about 8.
 - The pH is maintained by the balance between CO₂ and alkalinity level of the seawater.

$$CO_2 + CO_3^{2-} + H_2O \longleftrightarrow 2HCO_3^{-}$$

In a CO₂-rich ocean, the balance shifts to the right,
 i.e. ocean acidification.

$$[H^+] = K_2 \frac{[HCO_3^-]}{[CO_3^{2-}]} = K_2 \frac{2DIC - A_C}{A_C - DIC}$$

Spatial distributions

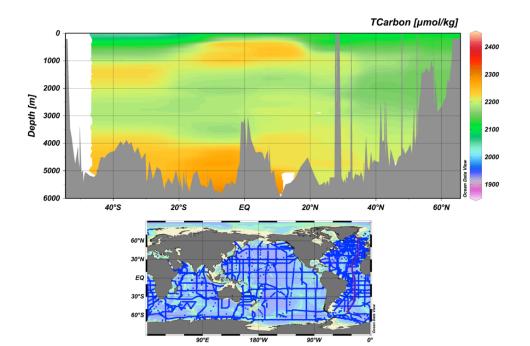


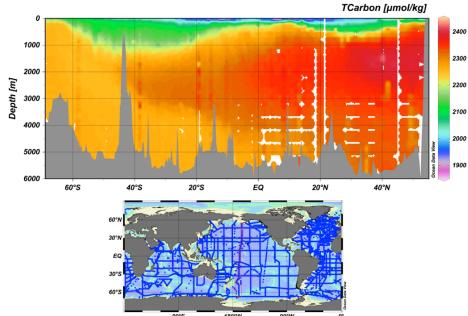
Williams_Fig. 6.4c,d

DIC in the oceans

Atlantic



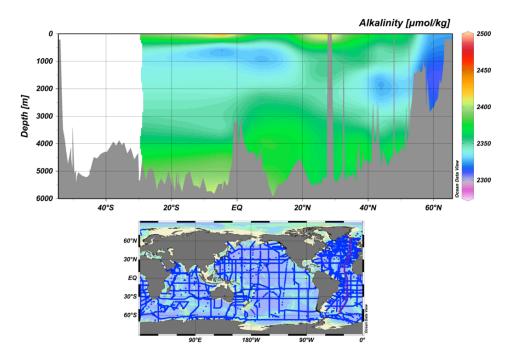




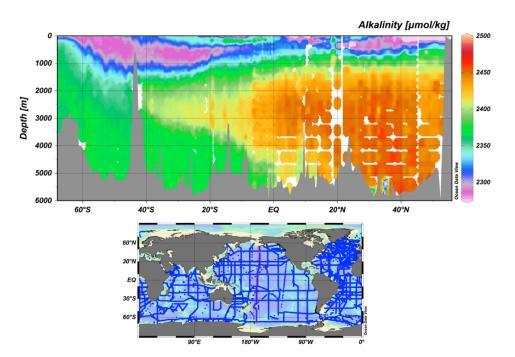
WOCE (1990s)

Alk in the oceans

Atlantic



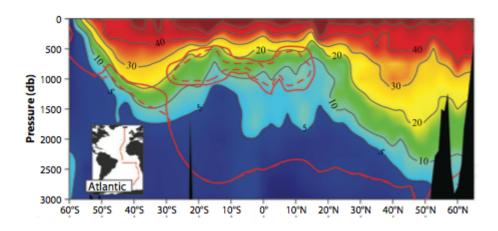
Pacific

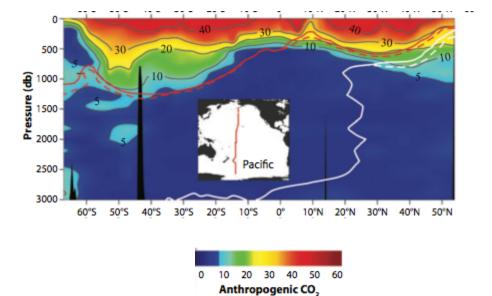


WOCE (1990s)

Anthropogenic CO₂

So far, the oceans absorbed about 1/3 of CO₂ emission The best estimate for the last few decades that ocean absorbs CO₂ at the rate of about 2 PgC/year. What regulates its spatial pattern? Most of anthropogenic CO₂ remains within the top 1,000m in the oceans. Why?



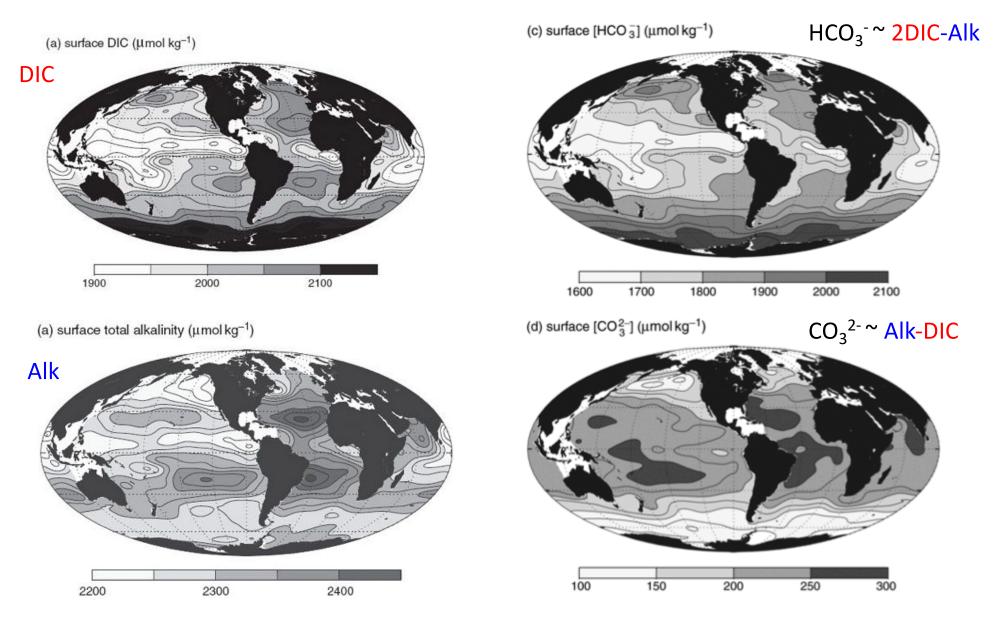


(umol ka-1)

Ocean carbon uptake

- Air-sea gas transfer
 Wind speed, solubility of CO₂
- Chemical reaction driving the CO_2 uptake: $CO_2 + CO_3^{2-} + H_2O \longleftrightarrow 2HCO_3^{-}$
 - Carbon uptake is chemically enhanced by this reaction. $[CO_3^{2-}]$ level controls its efficiency.

Spatial distributions



Williams_Fig. 6.4c,d

Is CO2 more soluble at high latitudes?

- Yes colder SST enhances the solubility of CO₂
- Why then tropics/subtropics contain more anthropogenic CO₂ in the surface water?
 - Subtropical water has higher alkalinity due to the excess evaporation

→ Enhanced buffering capacity (higher CO₃²⁻)
 → Subject to the same [CO₂] increase,
 subtropical water can absorb more carbon
 molecule than high latitudes

Revelle (Buffer) factor

- Equilibrium carbonate chemistry
 - Link between [CO₂] and DIC.
 - B is called Revelle (buffer) factor
 - B is about 10 in tropics/subtropics
 - B is about 18 in polar ocean
 - Anthropogenic CO₂ increase leads to different DIC response depending on the value of B

$$B \approx \frac{[HCO_3^{-}]}{[CO_3^{2-}]}$$

→ Spatial variation of both bicarbonate and carbonate ions increases B with latitude

$$\frac{\delta[CO_2]}{[CO_2]} = B \frac{\delta DIC}{DIC}$$

Some numbers

 Yearly rate of atmospheric CO2 increase fluctuates year to year, but it's magnitude is about 2ppm/year. Assuming saturation at the surface, can you predict the increase in the DIC of the surface water?

Use buffer factor,
$$\frac{\delta[CO_2]}{[CO_2]} = B \frac{\delta DIC}{DIC}$$

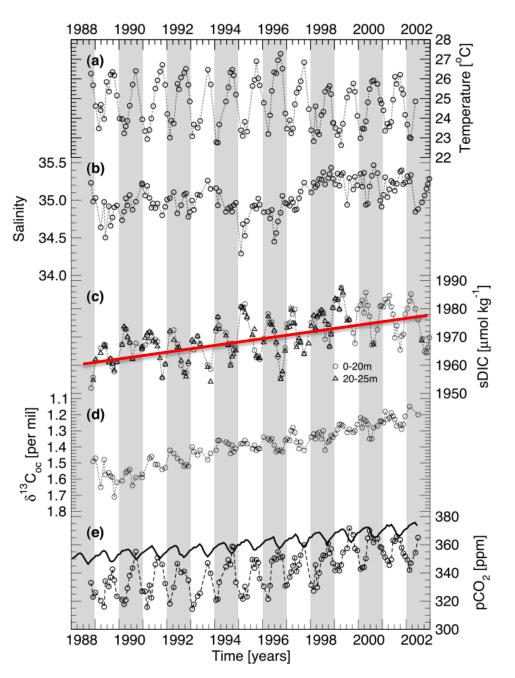
$$\begin{split} \delta DIC &= \frac{\delta [CO_2]}{B [CO_2]} DIC = \frac{\delta [CO_2]_{sat}}{B [CO_2]_{sat}} DIC = \frac{\delta p CO_{2atm}}{B p CO_{2atm}} DIC \\ &= \frac{2, ppm}{10 \times 400, ppm} \times 2000 \mu mol/kg = 1 \mu mol/kg \end{split}$$

Observed seasonal cycle in Hawaii

Long-term increase in oceanic DIC

Looking at the slope, the rate of increase is about 15 µmol/kg from 1988 to 2002

This is approximately about 1 µmol/kg per year!



More numbers

- If the surface ocean DIC is increasing at the rate of about 1 μ mol/kg per year for the surface mixed layer, what would be the yearly carbon uptake in PgC?
- Assume average MLD of 100m and the area of the global surface ocean is about 3.5 x 10¹⁴ m², we get:

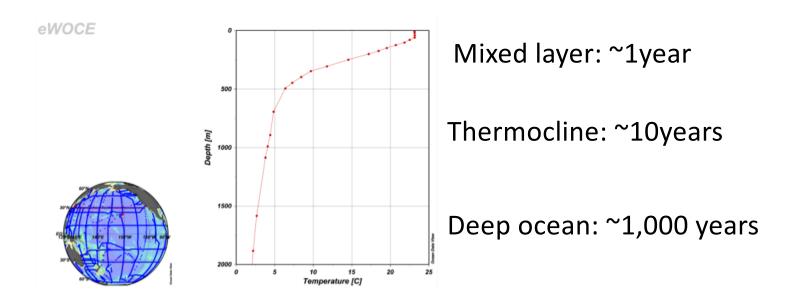
$$\rho V_{ML} \,\delta DIC = (1028 \, kgm^{-3}) \times (3.5 \cdot 10^{14} m^2 \times 100m) \times (1\mu mol/kg)$$

= $3.5 \cdot 10^{19} \mu mol = 3.5 \cdot 10^{13} mol = 4.3 \cdot 10^{14} gC = 0.43 PgC.$

 This is not enough to account for the ocean carbon uptake of 2 PgC/year.

Circulation, timescale of ventilation and the carbon sequestration

- Surface water is transported into the interior ocean
- Vertical circulation replaces subsurface water (via the ocean currents and mixing)
- Age of seawater ~ residence time of water

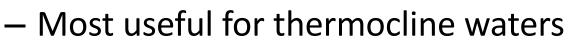


Water Age

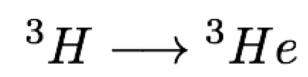
- How do we define the "age of water"?
 - How long has it been since the water was in last contact with the surface?
 - Age = 0 : surface water
 - Age generally increases with depth
- How can we measure it?

Observations of water age

- Carbon-14 (¹⁴C, radiocarbon)
 - Half life of 5,730 years
 - Natural C-14 is useful for deep waters
 - Bomb C-14 issues
- Tritium-Helium tracer
 - Half life of 12.43 years



- Transient tracers
 - CFC-11, -12, -113, SF₆ are typically used
 - Increasing trend in the atmosphere \rightarrow ocean
 - Ocean concentration lags behind the atmosphere
 - Most useful for thermocline waters



Observed C-14 age

• Matsumoto et al., 2007, JGR-Ocean

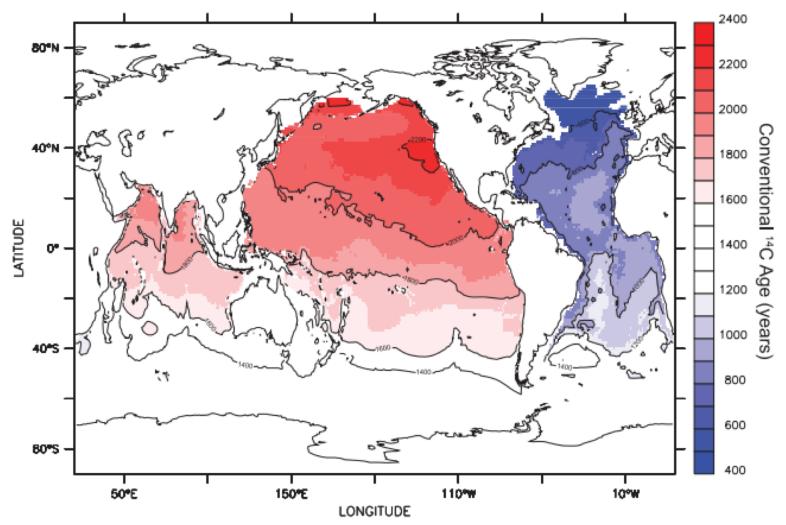


Figure 1. An objectively mapped conventional ¹⁴C age of natural radiocarbon below 1500 m, following *Matsumoto and Key* [2004]. Unit is years.

Issues with C-14 age

- The surface C-14 is not at equilibrium with the atmosphere
- Air-sea exchange of carbon isotope is very slow: it takes about 10 years to equilibrate the surface mixed layer → large air-sea disequilibrium
- This is called "surface reservoir age" indicating that the newly formed deep water has non-zero (positive) age.

Corrected C-14 age

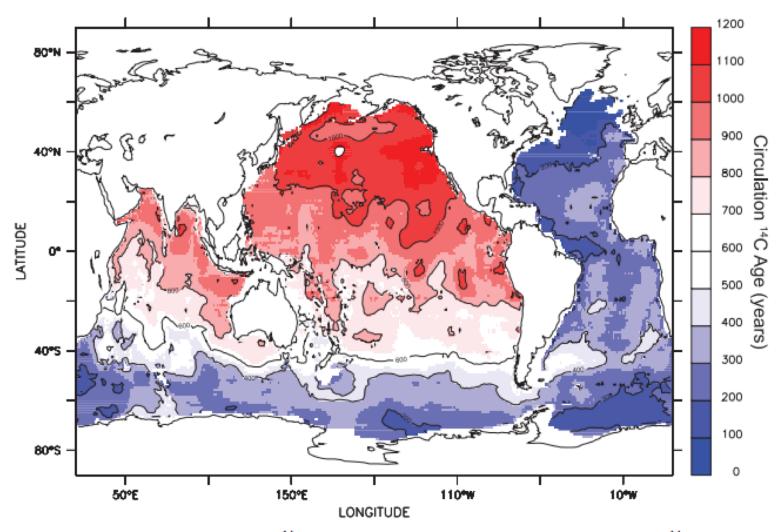
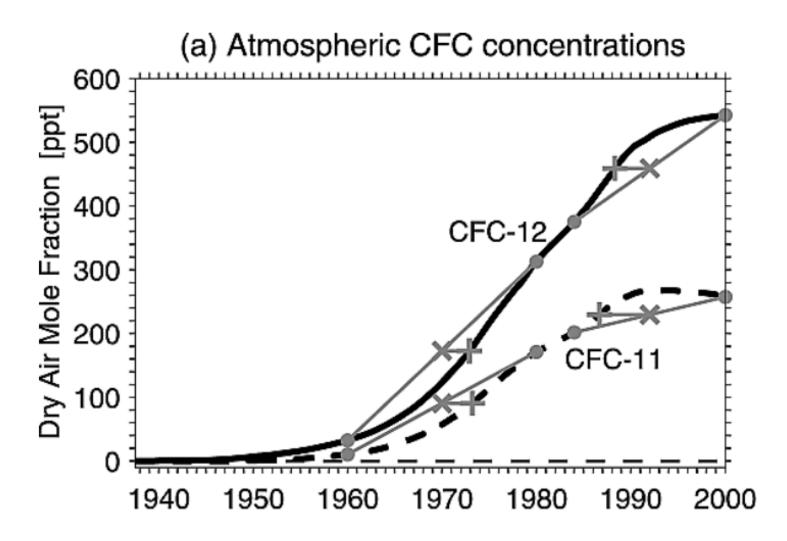


Figure 4. A map of circulation ¹⁴C age below 1500 m. This is equivalent to conventional ¹⁴C age (Figure 1) but accounts for surface ocean ¹⁴C reservoir age and the different sources of deep water. Unit is years.

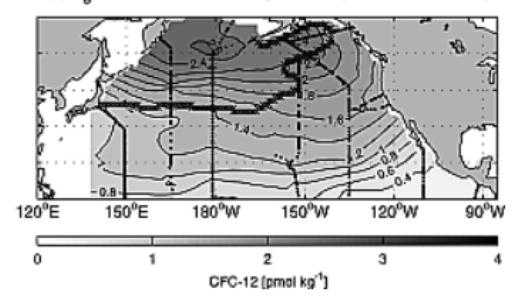
CFC-11, -12

• Mecking et al., (2004)

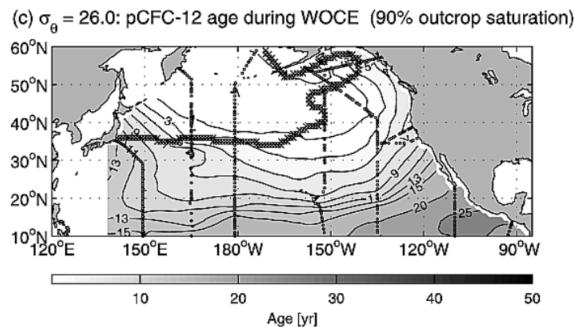


CFC-12 age in the thermocline

(d) σ_p = 26.0: CFC-12 during WOCE (corrected to 1993)



High CFC concentration means young age → more recently ventilated waters



The age of waters in the upper ocean thermocline < 10 years in mid-latitudes.

The effect of mixing tends to reduce the concentration age

Ocean carbon uptake

 As the ocean absorbs fossil fuel CO₂, what would happen to [HCO₃⁻] and [CO₃²⁻]?

– DIC increases but Alk remains the same.

$$[HCO_3^-] \sim 2DIC - Alk \uparrow$$
$$[CO_3^{2-}] \sim Alk - DIC \downarrow$$

Ocean carbon uptake

 If pCO₂ increases by 1%, what would be the DIC increase under constant Alk?

– Use the Buffer factor

$$\frac{\delta p C O_2}{p C O_2} = B \frac{\delta D I C}{D I C}$$

$$\frac{\delta DIC}{DIC} = \frac{1}{B} \times \frac{\delta p CO_2}{p CO_2} \sim \frac{1}{10}\%$$

Ocean carbon uptake

 As the ocean absorbs fossil fuel CO₂, what would happen to [H⁺] and Buffer factor?

- $[HCO_3^{-}]$ increases and $[CO_3^{2-}]$ decreases...

$$[H^+] = \frac{K_2[HCO_3^-]}{[CO_3^{2-}]} \uparrow$$

$$B = \frac{[HCO_3^{-}]}{[CO_3^{2-}]} = \frac{[H^+]}{K_2} \uparrow$$

Marine CaCO₃ production

- Calcifiers can biologically produce CaCO₃.
 (biomineralization)
- Impacts on DIC and Alkalinity
 - 1 mol of CaCO $_{\!3}$ production consumes 2 mols of alkalinity and 1 mol of DIC

$$Ca^{2+} + CO_3^{2-} \longrightarrow CaCO_3$$

- Implication for $[CO_3^{2-}]$ and $[HCO^{3-}]$?
- Implication for pH?

CaCO₃ dissolution

• Their dissolution is controlled by its solubility (Ω).

$$K_{SP} = [Ca^{2+}][CO_3^{2-}] \text{ at saturation}$$
$$\Omega = \frac{[Ca^{2+}][CO_3^{2-}]}{K_{SP}}$$

- If Ω is less than 1, it is undersaturated and the CaCO3 dissolves in the water.
- If Ω is greater than 1, it is supersaturated.

CaCO₃ dissolution

• Their dissolution is controlled by its solubility (Ω).

$$K_{SP} = [Ca^{2+}][CO_3^{2-}] \text{ at saturation}$$
$$\Omega = \frac{[Ca^{2+}][CO_3^{2-}]}{K_{SP}}$$

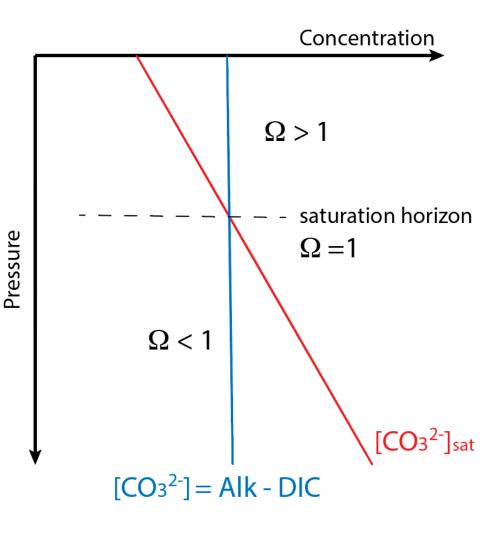
- Biogenic CaCO₃ has two mineral forms; calcite and aragonite (K_{SP} is different between the two)
- Aragonite is more unstable than calcite (larger Ksp)
- K_{SP} increases with pressure, so naturally deep waters tend to be undersaturated → CaCO₃ dissolves in the deep water.

Dissolution profile

$$[CO_3^{2-}]_{sat} \equiv \frac{K_{SP}}{[Ca^{2+}]}$$
$$\Omega = \frac{[CO_3^{2-}]}{[CO_3^{2-}]_{sat}}$$

The depths of saturation horizon ~ approx 3km in the Atlantic and 1km in the Pacific.

It is shoaling approx. 1-2 m/yr



Calcium carbonate pump

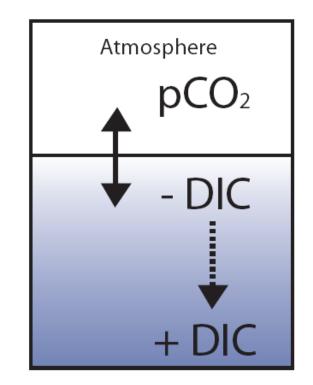
- Production of sinking CaCO3 particles at the surface
 - It removes both DIC and Alk from the surface water, lowering surface [CO₃²⁻].

$$CO_2 + CO_3^{2-} + H_2O \longleftrightarrow 2HCO_3^{-}$$

- The balance shifts to the left in the above equation \rightarrow increasing CO₂ in the surface water
- Surface water has more [CO₂], leading to degassing of CO2 to the atmosphere.

Carbon pumps

- Volk and Hoffert (1985)
 - Mechanisms that maintain vertical gradient of DIC
 in a steady state
 - Simple 1D view
 - Organic pump driven by sinking organic particle
 - Carbonate pump driven by sinking CaCO3 particles
 - Solubility pump driven by thermal stratification



Solubility pump

- Vertical temperature gradient
 - Global mean SST is about 15°C.
 - Deep water is about 2°C.
 - Deep water contains higher DIC relative to the surface
 - Considering the temperature dependence of solubility, the cooling of global ocean by 1°C leads to a lowering of atmospheric CO₂ by 10 ppm at steady state.

- Organic pump
 - Increases thermocline/deep nutrients and carbon.
 Also consumes oxygen.
 - Decreases atmospheric CO₂
- Carbonate (CaCO₃) pump
 - Increases thermocline/deep alkalinity and carbon.
 - Decreases buffering capacity at the surface, therefore increases atmospheric CO₂

Steady state carbon cycle

- Steady state $d/dt \rightarrow 0$
- Atmos + Ocean C inventory is constant.
- For the 1-box atmos 2-box ocean model;

Global C mass balance

$$M_a p C O_2^{atm} + (V_s C_s + V_d C_d) = (\text{constant})$$

Air-sea exch and carbonate chemistry

$$pCO_2^{ocn} = pCO_2^{atm}, \ C_s = f(pCO_2^{ocn}, Alk, K_i)$$

Deep ocean C balance

$$C_d = C_s + R_{C:P}(P_d - P_s)$$

Solubility pump

- Solubility pump acts through K₀ in the carbonate chemistry
- Look at a small perturbation

Global C mass balance

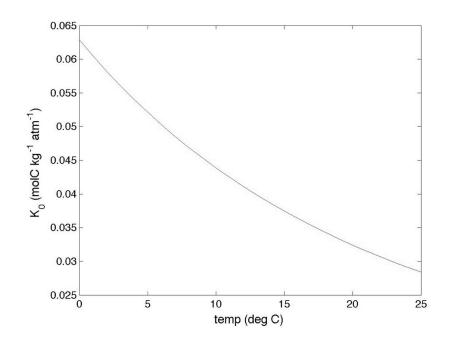
$$M_a \Delta p C O_2^{atm} + (V_s \Delta C_s + V_d \Delta C_d) = 0$$

Air-sea exch and carbonate chemistry

$$\Delta pCO_2^{ocn} = \Delta pCO_2^{atm}, \ \Delta C_s = g(\Delta pCO_2^{ocn}, \Delta K_0)$$

Deep ocean C balance

$$\Delta C_d = \Delta C_s$$



K_0 decreases with T

$$\frac{\partial K_0}{\partial T} \sim -0.0015 \left[Matm^{-1}K^{-1}\right]$$

With pCO2=280ppm,
Ma=1.8 x
$$10^{20}$$
 mol,
and K₀=0.04 we get
about 9ppm per 1°C
warming.

$$\Delta p C O_2 = -\frac{p C O_2}{\chi K_0} \Delta K_0$$

$$\chi = \frac{BM_a p CO_2 + VC}{VC} \sim 1.2$$

- The 2nd term in the deep ocean DIC
- Deep ocean P is essentially a constant, so Ps, is the only variable (i.e. *preformed P*).

Global C mass balance

$$M_a p C O_2^{atm} + (V_s C_s + V_d C_d) = (\text{constant})$$

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Global C mass balance

$$M_a \Delta p C O_2^{atm} + (V_s \Delta C_s + V_d \Delta C_d) = 0$$

Air-sea exch and carbonate chemistry

$$\Delta p CO_2^{ocn} = \Delta p CO_2^{atm}, \ \Delta C_s = f(\Delta p CO_2^{ocn})$$

Deep ocean C balance

$$\Delta C_d = \Delta C_s - R_{C:P} \Delta P_s$$

Ps, is measures the inefficiency of biological pump.
 Weaker productivity → higher surface P

$$\Delta p C O_2 = \frac{p C O_2 B R_{C:P}}{\chi C_s} \Delta P_s$$

With typical parameters, we get about 130ppm per $1\mu M$ increase in Ps

Transient carbon cycle

- Emission of fossil fuel CO₂ makes the carbon cycle off-balance.
- Modern C cycle is no longer in steady state.

 HW9 will be addressing what controls the rate of CO₂ uptake under increasing pCO₂^{atm}