EAS 4305/6305 Physics and Chemistry of the Oceans
Homework #3: Due before class, September 20th

Sea Level Rise (SLR) in the late 20th century

Tidal gauge measurements suggest that the globally averaged mean sea level has been rising since early 20th century at the rate of 1.5 to 2.0 [mm/year] (IPCC, 2001). More recently, satellite altimeter (available since early 1990s) has been used to further constrain the rate of sea level rise, suggesting a refined estimate of 1.8 ± 0.3 [mm/year] for 1950 to 2000 (Church et al., 2004). The rate of SLR is accelerating. More recent (1993-present) rate of SLR is more than 3 mm/year (NASA).

Early on, the thermal expansion of seawater due to the global warming has been considered as a primary driver of the sea level rise in earlier studies (e.g. IPCC, 1995). Levitus et al., (2005) determined that ocean heat content has increased by 14.5×10^{22} [J] from 1955 to 1998 in the top 3000 [m] of the global oceans.

(1) **Ocean warming**. Using above information, **estimate the global mean temperature increase** in the top 3000m for the period (1955-1998). The change in ocean heat content given by the following relationship.

$$(\text{change in heat content}) = \rho_0 c_P V \delta T [J]$$

V is the volume. You may assume the ocean covers 70% of the planet and the radius of the Earth is 6.3×10^6 m. The reference density (ρ_0) and the specific heat (c_P) of the seawater are 1025 [kgm^{-3}] and 3.9×10^3 [$JK^{-1}kg^{-1}$].

The impact of thermal expansion on the sea level is termed as the steric effect. The total mass of the ocean, M, can be written as $M = \rho V$, where ρ is the globally averaged density and V is the volume of the global ocean. The volume, V, can be further written as $V = A H$, where A is the surface area and H is the height of the global ocean. We will neglect the change of the surface area, so **the variation of sea level is linearly related to the volume change**.

(2) **Steric effect.** Assume that the mass of the ocean is conserved during the thermal expansion, ($\delta M=0$). Derive a mathematical expression for the steric sea level change (δHs) associated with density variation ($\delta\rho$).

(3) **Steric sea level rise between 1955-1998.** Using the results from (1) and (2), calculate the global steric height increase due to ocean warming (δHs), and convert it to the annual rate of sea level rise for the same period (in units of mm/year). You may use the linear equation of state: $\rho=\rho_0(1-\alpha\delta T+\beta\delta S)$ where $\alpha=2x10^{-4}(K^{-1})$ and $\beta=7x10^{-4}(psu^{-1})$.

(4) **Interpretation of data.** Compare the result of (3) to the satellite estimate of 1.8 ± 0.3 mm/year. Based on your answer, discuss whether the thermal expansion can be the primary cause of the late 20th century sea level change.

Another mechanism for the sea level change is the melting of the land ice and addition of freshwater to the oceans. This mechanism is termed as the eustatic effect. In this case, the mass of the ocean is no longer conserved.

(5) **Eustatic effect**. Based on the mass balance, derive a mathematical relationship between the melt mass input (δM) and the eustatic sea level change (δHe).

(6) **Ocean Mass budget** Assuming that global ocean warming estimates of Levitus et al., (2005) and the satellite observation of the net sea level rise are true, estimate the amount of melted land ice, δM [kg], required to close the ocean mass budget for the late 20th century.

(7) **Land Ice Mass budget** Approximately $26{,}350 \times 10^3$ km^3 of freshwater is stored as land ice (combining glaciers and ice sheets, Trenberth et al., 2007). Based on your answer to (6), estimate what fraction of land ice has melted during the late 20th century?

(8) **Projection** Based on what we learned in this exercise, discuss the most important factor in predicting future sea level rise? What information do we need to know in order to develop scenarios for the future sea level rise?

Cited references:

Church et al., (2004), Estimates of the Regional Distribution of Sea Level Rise over the 1950–2000 Period, J. Climate, Vol. 17, 2609-2625.

IPCC (1995), Climate change 1995: The science of climate change, Cambridge University Press, New York.

IPCC (2001), Climate change 2001: The scientific basis, Cambridge University Press, New York.

Levitus et al., (2005), Warming of the world ocean, 1995-2003, Geophys. Res. Lett., Vol. 32, L02604, doi:10.1029/2004GL021592.

Trenberth et al., (2007), Estimates of the Global Water Budget and Its Annual Cycle Using Observational and Model data, Journal of Hydrometeorology, 8,758, DOI: 10.1175/JHM600.1