EAS 4305/6305 Physics and Chemistry of the Oceans
Homework #3

Global warming and sea level rise

The thermal expansion of seawater due to the global warming has been considered as a primary driver of the sea level rise in earlier studies (e.g. IPCC, 1995). Levitus et al., (2005) determined that ocean heat content has increased by 14.5 x 10^{22} [J] from 1955 to 1998 in the top 3000 [m] of the global oceans.

(1) **Global Ocean Warming.** Using above information, **estimate the global mean temperature increase** in the top 3000m for the period (1955-1998). The change in ocean heat content given by the following relationship.

$$\text{(change in heat content)} = \rho_0 c_P V \delta T [J]$$

V is the volume. You may assume the ocean covers 70% of the planet and the radius of the Earth is 6.3 x 10^6 m. The reference density (ρ_0) and the specific heat (c_P) of the seawater are 1025 [kgm^{-3}] and 3.9 x 10^3 [$JK^{-1}kg^{-1}$].

V = 0.7 * 4 * 3.14 * (6.3 x 10^6)^2 * 3000 = 10^{18} (m^3)

δT = 14.5 x 10^{22} / (1025 * 3.9 x 10^3 * 10^{18}) = 0.0363 (K) increase from 1955 to 1998.

(2) **Steric effect.** Assume that the mass of the ocean is conserved during the thermal expansion, (δM=0). Derive a mathematical expression for the steric sea level change (δHs) associated with density variation ($\delta\rho$).

$$\delta H_S = \frac{\partial H}{\partial \rho}\delta\rho = -\frac{M}{\rho^2 A}\delta\rho = -H\frac{\delta\rho}{\rho}$$

(3) **Steric sea level rise between 1955-1998.** Using the results from (1) and (2), calculate the global steric height increase due to ocean warming (δHs), and convert it to the annual rate of sea level rise for the same period (in units of mm/year). You may use the linear equation of state: $\rho=\rho_0(1-\alpha\delta T+\beta\delta S)$ where $\alpha=2\text{x}10^{-4}(K^{-1})$ and $\beta=7\text{x}10^{-4}(psu^{-1})$.

Using linear equation of state, we get:

$$\delta H_S = -H\frac{\delta\rho}{\rho} = H\alpha\,\delta T$$

The steric sea level rise from 1955 to 1998 is:

3000 * 2x10^{-4} * 0.0363 (m) = 0.0218 (m).

This happened over the (1998-1955)=43 year period. Thus the annual rate of sea level rise is:

0.0218 / 43 (m/year) = 5.1 x 10^{-4} (m/year) = 0.51 (mm/year)

(4) **Interpretation of data.** Compare the result of (3) to the satellite estimate of 1.8 $\pm$ 0.3 mm/year. Based on your answer, discuss whether the thermal expansion can be the primary cause of the late 20th century sea level change.

Based on the observed temperature change, the steric sea level rise can only account for 24-33% of the observed sea level rise, this it is not the primary cause of the late 20th century sea level rise. The rest must be explained by other mechanisms.

(5) **Eustatic effect.** Based on the mass balance, derive a mathematical relationship between the melt mass input (δM) and the eustatic sea level change (δHe).

$$\delta H_e = \frac{\delta M}{\rho A} = H \frac{\delta M}{M}$$

(6) **Ocean Mass budget** Assuming that global ocean warming estimates of Levitus et al., (2005) and the satellite observation of the net sea level rise are true, estimate the amount of melted land ice, δM [kg], required to close the ocean mass budget for the late 20th century.

Based on (5) the mass change is linked to eustatic sea level rise as follows.

$$\delta M = M \frac{\delta H_e}{H}$$

M = 1025 (kg/m^3) * 10^{18} (m^3) = 10^{21} (kg). H is 3000m. Total sea level rise is 1.8 mm/yr and its steric component is 0.5 mm/yr. By taking the difference, the inferred eustatic component is 1.3 mm/yr. To close the mass budget, the yearly mass increase is inferred as:

10^{21} (kg) * 1.3 x 10^{-3} (m/year) / 3000(m) = 4.3 x 10^{14} (kg/year)

(7) **Land Ice Mass budget** Approximately 26,350x10^3 km^3 of freshwater is stored as land ice (Trenberth et al., 2007). Based on your answer to (6), estimate what fraction of land ice has melted during the late 20th century?

The land ice volume is

26,350 x 10^3 km^3 = 2.6 x $10^{(4+3+9)}$ = 2.6 x 10^{16} (m^3).

Then its mass is approximately 2.6 x 10^{19} (kg). Over the 43-year period, the estimated ocean mass increase is 4.3 x 10^{14} * 43 = 1.8 x 10^{16} (kg).

1.8 x 10^{16} / 2.6 x 10^{19} = 6.9 x 10^{-4}.

It is about 0.07% of total land ice.

- **Melting of glacier and ice sheets are the primary drivers of the global sea level rise, and is likely the dominant factor/uncertainty in the future of global sea level change.**